A-PDF Watermark DEMO: Purchase from www.A-PDF.com to remove the watermark

CUET UG JEE Mains NEET UG NCET

All these important exams is conducted by NTA. Very often some questions are twisted and repeated from these exams. So we include all the recent PYQs of all these exams to strenghten your CUET UG preparation.

Practicing these exam questions keeps you more relevant according to the question pattern of NTA.

NCET and CUET UG has same question pattern. Only difference is in the total number of questions being asked per subject.

Never buy or waste money on any mock test series because many mock test series are either self-made questions or old repeated or twisted PYQs of these exams or other entrance exams. Depending on those MOCK TEST SERIES may help you to practice questions but may keep you unprepared according to NTA's question pattern.

We have given some chapters for free to let you see what value are we providing you.

For all chapter pyqs question bank, please install our app from our website

www.cuet.pw

then purchase our question bank. Please support our hard work and effort. Please !!!!

Also check our Physics and Chemistry or others exam and subjects question bank.

Telegram Links

Just Search on Telegram

- @cuetpyq
- @books_cuet
- @class_12th_pyq
- @jee_cbse
- @pyqs12
- @cbsegroup
- @board_cbse

For CBSE Class 12 board exam chapter wise pyq question bank go to

www.cbse.page

www.cuet.pw

www.cbse.page

Chapter 7

Integrals

CUET 2025 PYQ 1

$\int_1^4 |x-2|dx$ is equal to

www.cuet.pw www.cbse.page

(a) 5 (b) $\frac{7}{2}$ (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

Concept Builder

www.cuet.pw www.cbse.page

Definite Integration of Modulus Function

www.cuet.pw www.cbse.page

Problem Statement

Evaluate the integral:

$$I = \int_1^4 |x-2|dx$$

www.cuet.pw www.cbse.page

Method 1: Splitting the Limits (Algebraic Method)

- **Concept:** The modulus function $|x-a|$ changes its definition at the critical point $x=a$.
- **Critical Point:** Here, $|x-2|$ changes behavior at $x=2$.
- Since the limit is from 1 to 4, the point $x=2$ lies inside our interval. We must split the integral at this point.
- **Definition Breakdown:**

$$|x-2| = \begin{cases} -(x-2) = 2-x & \text{if } x < 2 \\ x-2 & \text{if } x \geq 2 \end{cases}$$

- **Splitting the Integral:**

$$I = \int_1^2 (2-x)dx + \int_2^4 (x-2)dx$$

- **Integration:**

$$I_1 = \left[2x - \frac{x^2}{2}\right]_1^2$$

$$I_2 = \left[\frac{x^2}{2} - 2x\right]_2^4$$

- **Evaluating I_1 (Limit 1 to 2):**

$$I_1 = \left(2(2) - \frac{4}{2}\right) - \left(2(1) - \frac{1}{2}\right)$$

$$I_1 = (4 - 2) - (2 - 0.5)$$

$$I_1 = 2 - 1.5 = 0.5$$

- **Evaluating I_2 (Limit 2 to 4):**

$$I_2 = \left(\frac{16}{2} - 8\right) - \left(\frac{4}{2} - 4\right)$$

$$I_2 = (8 - 8) - (2 - 4)$$

$$I_2 = 0 - (-2) = 2$$

- **Total Integral:**

$$I = I_1 + I_2 = 0.5 + 2 = 2.5 = \frac{5}{2}$$

Correct Option: (d) $\frac{5}{2}$

www.cuet.pw www.cbse.page

Method 2: Geometric Area Method (Short Trick)

- **Concept:** The graph of $y = |x - 2|$ forms V-shaped triangles with the x-axis. The definite integral represents the area of these triangles.
- **Triangle 1 (Left of Vertex):**
  - Interval: From $x = 1$ to $x = 2$.
  - Base $= 2 - 1 = 1$.
  - Height (at $x = 1$) $= |1 - 2| = 1$.
  - $\text{Area}_1 = \frac{1}{2} \times 1 \times 1 = 0.5$.
- **Triangle 2 (Right of Vertex):**
  - Interval: From $x = 2$ to $x = 4$.

- ✅ Base $= 4 - 2 = 2$.
- ✅ Height (at $x = 4$) $= |4 - 2| = 2$.
- ✅ $\text{Area}_2 = \frac{1}{2} \times 2 \times 2 = 2$.

▷ **Total Area:** $0.5 + 2 = 2.5 = \frac{5}{2}$.

www.cuet.pw www.cbse.page

Method 3: The "Sum of Squares" Formula

▷ **Formula:** For $\int_a^b |x - c|\,dx$ where $a < c < b$:

$$\text{Area} = \frac{(c-a)^2 + (b-c)^2}{2}$$

▷ **Apply Values:** $a = 1, b = 4, c = 2$.

$$\text{Area} = \frac{(2-1)^2 + (4-2)^2}{2}$$

$$\text{Area} = \frac{1^2 + 2^2}{2}$$

$$\text{Area} = \frac{1+4}{2} = \frac{5}{2}$$

▷ This is the fastest method for objective exams.

CUET 2025 PYQ 2

The integral $I = \int \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{3\log_e x} - e^{2\log_e x}} dx$ is equal to

www.cuet.pw www.cbse.page

(1) $\frac{x}{2} + C$, where C is the constant of integration

(2) $\frac{x^2}{2} + C$, where C is the constant of integration

(3) $\frac{x^3}{3} + C$, where C is the constant of integration

(4) $\frac{x^4}{4} + C$, where C is the constant of integration

Concept Builder

www.cuet.pw www.cbse.page

Integration using Logarithmic Identities

www.cuet.pw www.cbse.page

Problem Statement

We need to evaluate the integral:

$$I = \int \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{3\log_e x} - e^{2\log_e x}} dx$$

www.cuet.pw www.cbse.page

Concept: The "Cancellation" Property

- **Property 1 (Power Rule):** $n\log_e x = \log_e(x^n)$.
- **Property 2 (Inverse Function Identity):** $e^{\log_e A} = A$.
- Since the exponential function (e^x) and natural logarithm ($\log_e x$) are inverse functions, they effectively cancel each other out.
- **Combined Formula:**

$$e^{n\log_e x} = e^{\log_e(x^n)} = x^n$$

Step-by-Step Solution

Step 1: Simplify the Terms

Using the formula $e^{n\log_e x} = x^n$, simplify every term in the expression:

- $e^{5\log_e x} = x^5$
- $e^{4\log_e x} = x^4$
- $e^{3\log_e x} = x^3$
- $e^{2\log_e x} = x^2$

Substitute these simplified values back into the integral:

$$I = \int \frac{x^5 - x^4}{x^3 - x^2} dx$$

Step 2: Factorize and Simplify

- Take out the highest common factor from the numerator (x^4) and the denominator (x^2):

$$I = \int \frac{x^4(x-1)}{x^2(x-1)} dx$$

- Cancel the common binomial factor $(x-1)$ and divide the powers of x:

$$I = \int \frac{x^4}{x^2} dx$$

$$I = \int x^{4-2} dx$$

$$I = \int x^2 dx$$

Step 3: Perform the Integration

- Apply the Power Rule of Integration: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.
- Here $n = 2$:

$$I = \frac{x^{2+1}}{2+1} + C$$

$$I = \frac{x^3}{3} + C$$

Correct Option: (3) $\frac{x^3}{3} + C$

Short Trick / Mental Method

- **Observation:** Just look at the coefficients of the log terms. They become the powers of x.
- **Mental Map:**

$$\frac{x^5 - x^4}{x^3 - x^2}$$

- Notice the gap between powers is same (1). Just divide the leading terms:

$$\frac{x^5}{x^3} = x^2$$

- Integrate x^2 immediately to get $\frac{x^3}{3}$.

CUET 2025 PYQ 3

$$\int_{\pi/6}^{\pi/3} \frac{\tan x}{\tan x + \cot x} dx \text{ is equal to}$$

www.cuet.pw www.cbse.page

(1) $\frac{\pi}{4}$ (2) 0 (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{12}$

Concept Builder

www.cuet.pw www.cbse.page

Definite Integration: King's Property

www.cuet.pw www.cbse.page

Problem Statement

Evaluate the integral:

$$I = \int_{\pi/6}^{\pi/3} \frac{\tan x}{\tan x + \cot x} dx$$

www.cuet.pw www.cbse.page

Method 1: Using King's Property (Standard Approach)

- **Concept:** The property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ is very useful when the integrand contains complementary trigonometric functions (like $\tan x$ and $\cot x$) and the limits sum to $\frac{\pi}{2}$.
- **Check Sum of Limits:**

$$a + b = \frac{\pi}{6} + \frac{\pi}{3} = 30^\circ + 60^\circ = 90^\circ = \frac{\pi}{2}$$

- **Step 1: Write the Original Integral**

$$I = \int_{\pi/6}^{\pi/3} \frac{\tan x}{\tan x + \cot x} dx \quad \ldots \text{(Equation 1)}$$

- **Step 2: Apply the Property** $x \to \left(\frac{\pi}{2} - x\right)$

$$I = \int_{\pi/6}^{\pi/3} \frac{\tan\left(\frac{\pi}{2} - x\right)}{\tan\left(\frac{\pi}{2} - x\right) + \cot\left(\frac{\pi}{2} - x\right)} dx$$

- Recall that $\tan(90^\circ - \theta) = \cot\theta$ and $\cot(90^\circ - \theta) = \tan\theta$.

$$I = \int_{\pi/6}^{\pi/3} \frac{\cot x}{\cot x + \tan x} dx \quad \ldots \text{(Equation 2)}$$

- **Step 3: Add Equation 1 and Equation 2**

$$2I = \int_{\pi/6}^{\pi/3} \left(\frac{\tan x}{\tan x + \cot x} + \frac{\cot x}{\tan x + \cot x} \right) dx$$

$$2I = \int_{\pi/6}^{\pi/3} \left(\frac{\tan x + \cot x}{\tan x + \cot x} \right) dx$$

- The numerator and denominator cancel out:

$$2I = \int_{\pi/6}^{\pi/3} 1 \, dx$$

- **Step 4: Evaluate the Integral**

$$2I = \left[x \right]_{\pi/6}^{\pi/3}$$

$$2I = \frac{\pi}{3} - \frac{\pi}{6}$$

$$2I = \frac{2\pi - \pi}{6} = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}$$

www.cuet.pw www.cbse.page

Method 2: Trigonometric Simplification

- **Step 1: Simplify the Integrand** Convert $\tan x$ and $\cot x$ into $\sin x$ and $\cos x$.

$$f(x) = \frac{\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$

Multiply numerator and denominator by $\sin x \cos x$:

$$f(x) = \frac{\sin^2 x}{\sin^2 x + \cos^2 x}$$

Since $\sin^2 x + \cos^2 x = 1$:

$$f(x) = \sin^2 x$$

- **Step 2: Integrate** $\sin^2 x$ Use the identity $\sin^2 x = \dfrac{1-\cos 2x}{2}$.

$$I = \int_{\pi/6}^{\pi/3} \frac{1-\cos 2x}{2} dx$$

$$I = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_{\pi/6}^{\pi/3}$$

- **Step 3: Apply Limits** Upper Limit $(\pi/3)$: $\dfrac{\pi}{3} - \dfrac{\sin(2\pi/3)}{2} = \dfrac{\pi}{3} - \dfrac{\sqrt{3}}{4}$ Lower Limit $(\pi/6)$: $\dfrac{\pi}{6} - \dfrac{\sin(\pi/3)}{2} = \dfrac{\pi}{6} - \dfrac{\sqrt{3}}{4}$
- **Subtract:**

$$I = \frac{1}{2}\left[\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) - \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)\right]$$

$$I = \frac{1}{2}\left[\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{1}{2}\left(\frac{\pi}{6}\right) = \frac{\pi}{12}$$

Correct Option: (4) $\dfrac{\pi}{12}$

www.cuet.pw www.cbse.page

Short Trick (The Half-Limit Rule)

- **Pattern:** If an integral is of the form:

$$\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} dx$$

(Here $f(x) = \tan x$ and $f(a+b-x) = \cot x$)

- **Result:** The answer is always half the difference of the limits.

$$\text{Answer} = \frac{b-a}{2}$$

- **Calculation:**

$$\frac{\frac{\pi}{3} - \frac{\pi}{6}}{2} = \frac{\frac{\pi}{6}}{2} = \frac{\pi}{12}$$

- This trick works for $\dfrac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$, $\dfrac{1}{1+\tan^n x}$, etc., provided $a+b = 90°$.

CUET 2025 PYQ 4

Match **List-I** with **List-II**

List-I (Definite integral)

(A) $\int_0^1 \frac{2x}{1+x^2}dx$

(B) $\int_{-1}^1 \sin^3 x \cos^4 x dx$

(C) $\int_0^\pi \sin x dx$

(D) $\int_2^3 \frac{2}{x^2-1}dx$

List-II (Value)

(I) 2

(II) $\log_e(3/2)$

(III) $\log_e 2$

(IV) 0

www.cuet.pw www.cbse.page

Choose the **correct** answer from the options given below:

(1) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

(2) (A) - (III), (B) - (II), (C) - (I), (D) - (IV)

(3) (A) - (III), (B) - (I), (C) - (IV), (D) - (II)

(4) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Concept Builder

www.cuet.pw www.cbse.page

Definite Integration: Matching & Solving

www.cuet.pw www.cbse.page

Problem Analysis

We need to evaluate four different definite integrals and match them with their values.

www.cuet.pw www.cbse.page

Step-by-Step Evaluation

(A) Evaluate $\int_0^1 \frac{2x}{1+x^2} dx$

- **Concept:** Integration by Substitution.
- Let $1 + x^2 = t$.
- Differentiating both sides: $2x\,dx = dt$.
- **Change Limits:** When $x = 0 \implies t = 1 + 0^2 = 1$. When $x = 1 \implies t = 1 + 1^2 = 2$.
- **Integral:**

$$I = \int_1^2 \frac{1}{t} dt$$

$$I = \left[\log_e t\right]_1^2$$

$$I = \log_e 2 - \log_e 1$$

- Since $\log_e 1 = 0$, we get $I = \log_e 2$.
- **Match:** (A) $\rightarrow$ (III)

(B) Evaluate $\int_{-1}^1 \sin^3 x \cos^4 x\, dx$

- **Concept:** Property of Definite Integrals for symmetric limits $[-a, a]$.
- Check if the function $f(x) = \sin^3 x \cos^4 x$ is Even or Odd.

$$f(-x) = \sin^3(-x)\cos^4(-x)$$

- Since $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$:

$$f(-x) = (-\sin x)^3(\cos x)^4 = -\sin^3 x \cos^4 x$$

$$f(-x) = -f(x)$$

- Since $f(x)$ is an **Odd Function**, the integral over $[-1, 1]$ is directly **zero**.
- **Match:** (B) $\rightarrow$ (IV)

(C) Evaluate $\int_0^\pi \sin x\, dx$

- **Concept:** Standard Integration.
- We know $\int \sin x\,dx = -\cos x$.

$$I = [-\cos x]_0^{\pi}$$

$$I = -(\cos\pi - \cos 0)$$

- Substitute values ($\cos\pi = -1, \cos 0 = 1$):

$$I = -(-1 - 1)$$

$$I = -(-2) = 2$$

- **Match:** (C) $\rightarrow$ (I)

(D) Evaluate $\displaystyle\int_2^3 \frac{2}{x^2-1}dx$

- **Concept:** Standard Formula $\displaystyle\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log_e\left|\frac{x-a}{x+a}\right|$.
- Here, the constant 2 is outside, and $a = 1$.

$$I = 2\int_2^3 \frac{dx}{x^2-1^2}$$

$$I = 2\left[\frac{1}{2(1)}\log_e\left|\frac{x-1}{x+1}\right|\right]_2^3$$

- The factor 2 cancels out:

$$I = \left[\log_e\left|\frac{x-1}{x+1}\right|\right]_2^3$$

- **Apply Limits:** Upper Limit ($x = 3$): $\log_e\left(\frac{2}{4}\right) = \log_e\left(\frac{1}{2}\right)$. Lower Limit ($x = 2$): $\log_e\left(\frac{1}{3}\right)$.

$$I = \log_e\left(\frac{1}{2}\right) - \log_e\left(\frac{1}{3}\right)$$

- Use $\log a - \log b = \log(a/b)$:

$$I = \log_e\left(\frac{1/2}{1/3}\right) = \log_e\left(\frac{3}{2}\right)$$

- **Match:** (D) $\rightarrow$ (II)

www.cuet.pw www.cbse.page

Final Matching

- (A) → (III)
- (B) → (IV)
- (C) → (I)
- (D) → (II)

Comparing with options: **Correct Option:** (4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

www.cuet.pw www.cbse.page

Short Trick / Visual Check

- **Visual Odd/Even Check:** Whenever you see limits like $-a$ to a (e.g., -1 to 1), **always** check for odd functions first. Odd powers of sine ($\sin^3 x, \sin^5 x$) multiplied by any power of cosine are always odd. This instantly solves part (B) as 0 without any calculation.
- **Area under Sine:** The area under one "hump" of $\sin x$ (from 0 to π) is a standard result worth memorizing: it is always **2**.

CUET 2025 PYQ 5

The integral $I = \int e^x \left(\frac{x-1}{3x^2}\right) dx$ is equal to

www.cuet.pw www.cbse.page

(1) $\frac{1}{3}\left(\frac{x^2}{2} - x\right) + C$, where C is constant of integration

(2) $\left(\frac{x^2}{2} - x\right) e^x + C$, where C is constant of integration

(3) $\frac{1}{3x^2} e^x + C$, where C is constant of integration

(4) $\frac{1}{3x} e^x + C$, where C is constant of integration

Concept Builder

www.cuet.pw www.cbse.page

Integration by Parts: Standard Exponential Form

www.cuet.pw www.cbse.page

Problem Statement

We need to evaluate the integral:

$$I = \int e^x \left(\frac{x-1}{3x^2}\right) dx$$

www.cuet.pw www.cbse.page

Concept: The Standard Form

- **The Golden Rule of Exponential Integration:** Whenever you see an integral of the form $\int e^x [f(x) + f'(x)] dx$, the answer is always $e^x f(x) + C$.
- **Why?** Because of the Product Rule of differentiation:

$$\frac{d}{dx}[e^x f(x)] = e^x f(x) + e^x f'(x) = e^x [f(x) + f'(x)]$$

- **Goal:** We must manipulate the algebraic part inside the bracket $\left(\frac{x-1}{3x^2}\right)$ to look like a sum of a function and its derivative.

www.cuet.pw www.cbse.page

Step-by-Step Solution

Step 1: Simplify the Algebraic Expression

- Separate the terms in the numerator:

$$\frac{x-1}{3x^2} = \frac{x}{3x^2} - \frac{1}{3x^2}$$

- Simplify the fractions:

$$= \frac{1}{3x} - \frac{1}{3x^2}$$

- Factor out the constant $\frac{1}{3}$ to make it cleaner:

$$= \frac{1}{3}\left(\frac{1}{x} - \frac{1}{x^2}\right)$$

Step 2: Identify $f(x)$ and $f'(x)$

- Substitute the simplified expression back into the integral:

$$I = \frac{1}{3}\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

- Let's test the terms: Let $f(x) = \frac{1}{x} = x^{-1}$.
- Differentiate $f(x)$:

$$f'(x) = -1 \cdot x^{-2} = -\frac{1}{x^2}$$

- Now look at the expression inside the integral:

$$\left(\frac{1}{x} + \left(-\frac{1}{x^2}\right)\right)$$

This matches the form $[f(x) + f'(x)]$ perfectly.

Step 3: Apply the Formula

- Using the standard result $\int e^x [f(x) + f'(x)]\,dx = e^x f(x)$:

$$I = \frac{1}{3}\left[e^x \cdot f(x)\right] + C$$

- Substitute $f(x) = \dfrac{1}{x}$:

$$I = \frac{1}{3}\left[e^x \cdot \frac{1}{x}\right] + C$$

$$I = \frac{e^x}{3x} + C$$

Correct Option: (4) $\dfrac{1}{3x}e^x + C$

www.cuet.pw www.cbse.page

Short Trick / Pattern Recognition

- **Mental Step 1:** Pull out the constant 1/3 immediately.

$$\frac{1}{3}\int e^x \left(\frac{x-1}{x^2}\right)$$

- **Mental Step 2:** Split the fraction.

$$\frac{x}{x^2} - \frac{1}{x^2} = \frac{1}{x} - \frac{1}{x^2}$$

- **Mental Step 3:** Recognize the pair. Derivative of $\dfrac{1}{x}$ is $-\dfrac{1}{x^2}$.
- **Result:** The answer is always $e^x \times$ (The Function Term).

$$\text{Answer} = \frac{1}{3}e^x\left(\frac{1}{x}\right)$$

- This entire process takes less than 10 seconds once you recognize the $f(x) + f'(x)$ structure.

NCET 2025 PYQ 6

$$\int \frac{1}{\sqrt{x^2+6x-7}}dx \text{ is equal to :}$$

www.cuet.pw www.cbse.page

(a) $\log_e\left|(x-3)+\sqrt{x^2+6x-7}\right|+C$; where C is constant of integration

(b) $\frac{1}{4}\log_e\left|(x-3)+\sqrt{x^2+6x-7}\right|+C$; where C is constant of integration

(c) $\log_e\left|(x+3)+\sqrt{x^2+6x-7}\right|+C$; where C is constant of integration

(d) $\frac{1}{4}\log_e\left|(x+3)+\sqrt{x^2+6x-7}\right|+C$; where C is constant of integration

Concept Builder

www.cuet.pw www.cbse.page

1. Core Concepts

- **Completing the Square:** The standard method for integrating $\frac{1}{\sqrt{ax^2+bx+c}}$ is to rewrite the quadratic expression inside the square root in the form $(x+A)^2 \pm B^2$.
- **Standard Integral Formula:** Once in the form $(u)^2 - a^2$, we use the formula:

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \log_e\left|x+\sqrt{x^2-a^2}\right|+C$$

(Note: Sometimes written as $\ln$ *instead of* $\log_e$*)*

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

(1) **Step 1: Complete the Square for the Denominator** Consider the quadratic expression inside the radical:

$$Q = x^2+6x-7$$

To complete the square for x^2+6x:

- Take the coefficient of x, which is 6.
- Halve it: $\frac{6}{2} = 3$.
- Square it: $3^2 = 9$.

Add and subtract 9 inside the expression:

$$x^2 + 6x + 9 - 9 - 7$$

$$(x^2 + 6x + 9) - 16$$

$$(x+3)^2 - 4^2$$

(2) Step 2: Rewrite the Integral Substitute the completed square form back into the integral:

$$I = \int \frac{1}{\sqrt{(x+3)^2 - 4^2}} dx$$

(3) Step 3: Apply the Integration Formula Let $u = x + 3$, then $du = dx$. The integral becomes:

$$I = \int \frac{du}{\sqrt{u^2 - 4^2}}$$

Using the standard formula $\int \frac{du}{\sqrt{u^2 - a^2}} = \log_e |u + \sqrt{u^2 - a^2}|$:

$$I = \log_e \left| (x+3) + \sqrt{(x+3)^2 - 4^2} \right| + C$$

(4) Step 4: Simplify back to Original Form We know that $(x+3)^2 - 16$ is just the rewritten form of $x^2 + 6x - 7$. So, replace the term inside the square root:

$$I = \log_e \left| (x+3) + \sqrt{x^2 + 6x - 7} \right| + C$$

www.cuet.pw www.cbse.page

3. Short Tricks (Pattern Matching)

- **Identify the "Linear Part":** When you complete the square for $x^2 + 6x$, the linear term inside the bracket is always $(x + \frac{b}{2})$. Here $b = 6$, so the term must be $(x + 3)$.
- **Check the Options:**

- Options (a) and (b) have $(x-3)$. **Incorrect**.
- Options (c) and (d) have $(x+3)$. **Correct**.

- **Check for Coefficients:** The standard formula for $\frac{1}{\sqrt{x^2-a^2}}$ does **not** have a fraction like $1/4$ or $1/2a$ outside the log. (That fraction appears for integrals without square roots, like $\frac{1}{x^2-a^2}$).
- Option (d) has a $\frac{1}{4}$ coefficient. **Incorrect**.
- Option (c) has coefficient 1. **Correct**.

- This leaves only option (c).

www.cuet.pw www.cbse.page

4. Final Answer

- The integral is equal to $\log_e \left|(x+3)+\sqrt{x^2+6x-7}\right| + C$.
- This corresponds to option **(c)**.

NCET 2025 PYQ 7

If a, b and c are some real constants, then the value of the integral $\int_{-2}^{2} (ax^3 + bx + c)\, dx$ depends on :

www.cuet.pw www.cbse.page

(a) both a and b (b) both b and c (c) all a, b and c (d) c only

Concept Builder

www.cuet.pw www.cbse.page

1. Core Concepts

- **Symmetric Limits Property:** When integrating over an interval $[-k, k]$ (symmetric about the origin), we classify the integrand into Odd and Even functions.
- **Odd Function:** A function where $f(-x) = -f(x)$. (e.g., $x^1, x^3, \sin x$). **Property:** $\int_{-k}^{k} f_{\text{odd}}(x)\, dx = 0$
- **Even Function:** A function where $f(-x) = f(x)$. (e.g., constants, $x^2, \cos x$). **Property:** $\int_{-k}^{k} f_{\text{even}}(x)\, dx = 2\int_{0}^{k} f(x)\, dx$

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

(1) Step 1: Split the Integral We can separate the integral into three parts based on the terms:

$$I = \int_{-2}^{2} (ax^3 + bx + c)\, dx$$

$$I = \int_{-2}^{2} ax^3\, dx + \int_{-2}^{2} bx\, dx + \int_{-2}^{2} c\, dx$$

(2) Step 2: Analyze Each Term for Odd/Even Nature

- **Term 1 (ax^3):** The function $f(x) = x^3$ is Odd because $(-x)^3 = -x^3$.

$$\int_{-2}^{2} ax^3\, dx = 0 \quad \text{(Independent of } a\text{)}$$

- **Term 2 (bx):** The function $f(x) = x$ is Odd because $(-x) = -x$.

$$\int_{-2}^{2} bx\,dx = 0 \quad \text{(Independent of } b\text{)}$$

- **Term 3 (c):** A constant function $f(x) = c$ is Even because $f(-x) = c = f(x)$.

$$\int_{-2}^{2} c\,dx \neq 0$$

(3) Step 3: Evaluate the Remaining Integral The only surviving term is the constant c.

$$I = 0 + 0 + \int_{-2}^{2} c\,dx$$

$$I = [cx]_{-2}^{2}$$

$$I = c(2) - c(-2)$$

$$I = 2c + 2c = 4c$$

www.cuet.pw www.cbse.page

3. Conclusion and Short Trick

- **Result:** The value of the integral is $4c$.
- Since a and b disappeared from the calculation (because their terms integrated to zero), the final answer depends **only on** c.
- **Short Trick:** "Symmetric limits kill odd powers." Mentally cross out ax^3 and bx. You are left with only c.

www.cuet.pw www.cbse.page

4. Final Answer

- The value of the integral depends on c **only**.
- This corresponds to option **(d)**.

NCET 2025 PYQ 8

The value of $\int_0^{\pi} \frac{1}{1+e^{\cos x}} dx$, is :

www.cuet.pw www.cbse.page

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $-\pi$

Concept Builder

www.cuet.pw www.cbse.page

1. Core Concepts

- **King's Property of Definite Integrals:** This is the most powerful tool for integrals involving limits.

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

For limits 0 to π, this becomes:

$$\int_0^{\pi} f(x)\,dx = \int_0^{\pi} f(\pi-x)\,dx$$

- **Trigonometric Identity:**

$$\cos(\pi - x) = -\cos x$$

- **Algebraic Identity for Exponentials:**

$$\frac{1}{1+e^{-u}} = \frac{1}{1+\frac{1}{e^u}} = \frac{e^u}{e^u+1}$$

Crucially, adding the original term and the transformed term sums to 1:

$$\frac{1}{1+e^u} + \frac{e^u}{1+e^u} = \frac{1+e^u}{1+e^u} = 1$$

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

(1) Step 1: Let the Integral be I

$$I = \int_0^{\pi} \frac{1}{1 + e^{\cos x}} \, dx \quad \ldots \text{(Equation 1)}$$

(2) Step 2: Apply King's Property Replace x with $(\pi - x)$:

$$I = \int_0^{\pi} \frac{1}{1 + e^{\cos(\pi - x)}} \, dx$$

Using $\cos(\pi - x) = -\cos x$:

$$I = \int_0^{\pi} \frac{1}{1 + e^{-\cos x}} \, dx$$

(3) Step 3: Simplify the New Equation Multiply numerator and denominator by $e^{\cos x}$ to remove the negative exponent:

$$I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + 1} \, dx \quad \ldots \text{(Equation 2)}$$

(4) Step 4: Add Equation (1) and Equation (2)

$$I + I = \int_0^{\pi} \frac{1}{1 + e^{\cos x}} \, dx + \int_0^{\pi} \frac{e^{\cos x}}{1 + e^{\cos x}} \, dx$$

$$2I = \int_0^{\pi} \left(\frac{1 + e^{\cos x}}{1 + e^{\cos x}} \right) dx$$

$$2I = \int_0^{\pi} 1 \, dx$$

(5) Step 5: Evaluate the Simple Integral

$$2I = [x]_0^{\pi}$$

$$2I = \pi - 0$$

$$2I = \pi$$

Solve for I:

$$I = \frac{\pi}{2}$$

www.cuet.pw www.cbse.page

3. Short Tricks (The "Half-Interval" Rule)

- **Pattern Recognition:** Whenever you see an integral of the form:

$$\int_0^a \frac{1}{1+e^{f(x)}}\,dx$$

  where $f(a-x) = -f(x)$ (i.e., the function inside the exponent is odd with respect to the midpoint), the answer is **always half the length of the interval**.

- **Application:**
  - ✅ Interval length $= \pi - 0 = \pi$.
  - ✅ Function $\cos x$ satisfies the symmetry property $\cos(\pi - x) = -\cos x$.
  - ✅ **Result:** Answer $= \dfrac{\text{Length}}{2} = \dfrac{\pi}{2}$.
- This trick works for functions like $\sin x$ over 0 to 2π, or just x over $-a$ to a.

www.cuet.pw www.cbse.page

4. Final Answer

- The value of the integral is $\dfrac{\pi}{2}$.
- This corresponds to option **(b)**.

NCET 2025 PYQ 9

$\int \frac{\cos(2x)}{(\sin(x)-\cos(x))^3}\,dx$ is equal to :

www.cuet.pw www.cbse.page

(a) $\log_e|\cos(x)-\sin(x)|+C$

(b) $\log_e|\cos(x)+\sin(x)|+C$

(c) $\frac{-1}{\cos(x)-\sin(x)}+C$

(d) $\frac{-1}{(\cos(x)-\sin(x))^2}+C$

Concept Builder

www.cuet.pw www.cbse.page

1. Core Concepts

- **Double Angle Identity:** The key to unlocking this problem is the identity for $\cos(2x)$.

$$\cos(2x)=\cos^2 x-\sin^2 x$$

This can be factored as a difference of squares:

$$\cos(2x)=(\cos x-\sin x)(\cos x+\sin x)$$

- **u-Substitution:** A common integration technique where we choose a part of the integrand to be u, such that its derivative du is present in the numerator (or easily manipulatable).

$$\int f(g(x))g'(x)dx=\int f(u)du$$

- **Power Rule for Integration:**

$$\int u^n du=\frac{u^{n+1}}{n+1}+C \quad (\text{for } n\neq -1)$$

Specifically for $n = -2$:

$$\int \frac{1}{u^2} du = \int u^{-2} du = \frac{u^{-1}}{-1} = -\frac{1}{u}$$

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

(1) Step 1: Expand the Numerator We start with the integral:

$$I = \int \frac{\cos(2x)}{(\sin x - \cos x)^3} dx$$

Using the identity $\cos(2x) = \cos^2 x - \sin^2 x$:

$$I = \int \frac{\cos^2 x - \sin^2 x}{(\sin x - \cos x)^3} dx$$

Factor the numerator as a difference of squares:

$$I = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x - \cos x)^3} dx$$

(2) Step 2: Simplify the Fraction Notice the terms $(\cos x - \sin x)$ and $(\sin x - \cos x)$. They are negatives of each other.

$$(\cos x - \sin x) = -(\sin x - \cos x)$$

Substitute this back:

$$I = \int \frac{-(\sin x - \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^3} dx$$

Cancel one factor of $(\sin x - \cos x)$ from the numerator and denominator:

$$I = \int \frac{-(\cos x + \sin x)}{(\sin x - \cos x)^2} dx$$

Rearrange the negative sign into the denominator (squared term) or keep it outside. Let's keep it outside for clarity, but note that $(\sin x - \cos x)^2 = (\cos x - \sin x)^2$. Let's switch the denominator to match the derivative better later.

$$I = -\int \frac{\sin x + \cos x}{(\sin x - \cos x)^2} dx$$

(3) **Step 3: Apply Substitution** Let $u = \sin x - \cos x$. Differentiate with respect to x:

$$du = (\cos x - (-\sin x))\,dx$$

$$du = (\cos x + \sin x)\,dx$$

This matches the numerator exactly!

(4) **Step 4: Integrate in terms of u** Substitute u and du into the integral:

$$I = -\int \frac{1}{u^2} du$$

$$I = -\int u^{-2} du$$

Using the power rule:

$$I = -\left(\frac{u^{-2+1}}{-2+1}\right) + C$$

$$I = -\left(\frac{u^{-1}}{-1}\right) + C$$

$$I = u^{-1} + C = \frac{1}{u} + C$$

(5) **Step 5: Back-Substitute** Replace u with $\sin x - \cos x$:

$$I = \frac{1}{\sin x - \cos x} + C$$

Wait, let's check the options. Option (c) is $\frac{-1}{\cos x - \sin x}$. Note that $\frac{1}{\sin x - \cos x} = \frac{1}{-(\cos x - \sin x)} = \frac{-1}{\cos x - \sin x}$.

This matches perfectly.

www.cuet.pw www.cbse.page

3. Short Tricks (Pattern Recognition)

- **Degree Check:** Look at the powers. Numerator degree effectively 2 (trig squared). Denominator degree 3. Net degree = $2 - 3 = -1$ (in terms of trig functions). When integrated, power rule ($x^n \to x^{n+1}$) suggests the answer should have power $-1 + 1 = 0$ (Logarithm) OR if we simplify first, power $-2 \to -1$.

- **The "Derivative Pair" Trick:** Always check if the numerator is related to the derivative of the base of the denominator. Base: $\sin x - \cos x$. Derivative: $\cos x + \sin x$. Numerator $\cos 2x$ contains $(\cos x + \sin x)$ as a factor. So, one factor cancels, reducing the denominator power from 3 to 2. Integrating $1/u^2$ gives $-1/u$. This immediately points to an answer of the form $\frac{k}{\text{Base}^1}$. Only option (c) has the Base to the power 1 in the denominator. Options (a) and (b) are logs, Option (d) is squared.

www.cuet.pw www.cbse.page

4. Final Answer

- The integral is equal to $\frac{-1}{\cos x - \sin x} + C$.
- This corresponds to option **(c)**.

NCET 2025 PYQ 10

If $[x]$ denotes the greatest integer function, then $\int_0^3 [x]\,dx$ is equal to :

www.cuet.pw www.cbse.page

(a) 0 (b) 1 (c) 2 (d) 3

Concept Builder

www.cuet.pw www.cbse.page

Concept: The Greatest Integer Function

- The notation $[x]$ represents the **Greatest Integer Function** (also known as the floor function).
- It returns the largest integer that is less than or equal to x.
- **Key Values:**
  - If $0 \le x < 1$, then $[x] = 0$.
  - If $1 \le x < 2$, then $[x] = 1$.
  - If $2 \le x < 3$, then $[x] = 2$.
- **Integration Rule:** Since $[x]$ is a discontinuous step function (it jumps at every integer), we cannot integrate it directly over the whole range $[0,3]$. We must use the property of definite integrals to split the limits at every integer.

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

www.cuet.pw www.cbse.page

Method 1: Detailed Solution (Splitting Limits)

- The integration range is from 0 to 3. We break this into sub-intervals where the value of $[x]$ is constant: $(0 \to 1)$, $(1 \to 2)$, and $(2 \to 3)$.

$$I = \int_0^3 [x]\,dx = \int_0^1 [x]\,dx + \int_1^2 [x]\,dx + \int_2^3 [x]\,dx$$

- Substitute the value of $[x]$ for each specific interval:

$$I = \int_0^1 0 \cdot dx + \int_1^2 1 \cdot dx + \int_2^3 2 \cdot dx$$

- Evaluate the simple integrals. Recall that $\int k\,dx = kx$.

$$I = [0]_0^1 + [x]_1^2 + [2x]_2^3$$

- Apply the limits: Upper Limit - Lower Limit.

$$I = 0 + (2-1) + 2(3-2)$$

$$I = 0 + 1 + 2(1)$$

$$I = 1 + 2 = 3$$

www.cuet.pw www.cbse.page

Method 2: Graphical Method (Short Trick)

- The definite integral represents the **Area Under the Curve**.
- The graph of $y = [x]$ looks like a staircase.
- We can simply calculate the area of the rectangles formed under the graph between $x = 0$ and $x = 3$.
- **Rectangle 1 (x from 0 to 1):** Height is 0. Area = $1 \times 0 = 0$.
- **Rectangle 2 (x from 1 to 2):** Height is 1. Area = $1 \times 1 = 1$.
- **Rectangle 3 (x from 2 to 3):** Height is 2. Area = $1 \times 2 = 2$.

$$\text{Total Area} = 0 + 1 + 2 = 3$$

www.cuet.pw www.cbse.page

Method 3: Direct Formula (Super Short Trick)

- If you need to solve $\int_0^n [x]\,dx$ where n is a positive integer, the answer is simply the sum of integers from 0 to $n-1$.
- **Formula:**

$$\int_0^n [x]\,dx = \frac{n(n-1)}{2}$$

▷ **Execution:** Here, $n = 3$.

$$I = \frac{3\,(3-1)}{2} = \frac{3 \times 2}{2} = 3$$

www.cuet.pw www.cbse.page

Final Answer

$$\int_0^3 [x]\,dx = 3$$

NCET 2025 PYQ 11

$\int \frac{dx}{1+e^{-5x}}$ is equal to :

www.cuet.pw www.cbse.page

(a) $\frac{1}{5}\log_e\left|e^{-5x}\right|+C$

(b) $\frac{1}{5}\log_e\left|e^{5x}\right|+C$

(c) $\frac{1}{5}\log_e\left|1+e^{-5x}\right|+C$

(d) $\frac{1}{5}\log_e\left|1+e^{5x}\right|+C$

Concept Builder

www.cuet.pw www.cbse.page

Introduction to the Problem

We need to solve the following indefinite integral:

$$I=\int \frac{dx}{1+e^{-5x}}$$

- This is a standard integral type involving exponential functions in the denominator.
- Direct integration is difficult because the derivative of the denominator is not present in the numerator in its current form.
- We will use algebraic manipulation to transform the integrand into a form suitable for the method of substitution.

www.cuet.pw www.cbse.page

Method 1: The Standard Algebraic Approach (Detailed Steps)

This is the most reliable method for solving integrals of the form

$$\int \frac{dx}{1\pm e^{-ax}}$$

www.cuet.pw www.cbse.page

Step 1: Eliminate the Negative Exponent

- The term e^{-5x} can be difficult to work with directly.
- To simplify, we multiply both the numerator and the denominator by e^{5x}. This is effectively multiplying by $\left(\frac{e^{5x}}{e^{5x}}\right)$, which is equal to 1.

$$I = \int \frac{dx}{1+e^{-5x}} \cdot \frac{e^{5x}}{e^{5x}}$$

- Distribute e^{5x} in the denominator:

$$I = \int \frac{e^{5x}\,dx}{e^{5x}\cdot 1 + e^{5x}\cdot e^{-5x}}$$

- Recall that $e^{5x}\cdot e^{-5x} = e^0 = 1$. The integral becomes:

$$I = \int \frac{e^{5x}}{e^{5x}+1}\,dx$$

www.cuet.pw www.cbse.page

Step 2: Apply Integration by Substitution

- Now, notice that the numerator $\left(e^{5x}\right)$ is closely related to the derivative of the denominator $\left(e^{5x}+1\right)$.
- Let us define a new variable u:

$$\text{Let } u = e^{5x}+1$$

- Differentiate u with respect to x:

$$\frac{du}{dx} = \frac{d}{dx}\left(e^{5x}+1\right)$$

$$\frac{du}{dx} = 5e^{5x}$$

- Rearrange to solve for the term in our numerator ($e^{5x}dx$):

$$du = 5e^{5x}\,dx \implies \frac{du}{5} = e^{5x}\,dx$$

www.cuet.pw www.cbse.page

Step 3: Substitute and Solve

- Replace $\left(e^{5x}+1\right)$ with u and $\left(e^{5x}\,dx\right)$ with $\left(\frac{du}{5}\right)$ in the integral:

$$I=\int \frac{1}{u}\cdot\frac{du}{5}$$

$$I=\frac{1}{5}\int \frac{du}{u}$$

- Apply the standard integration formula $\int \frac{dx}{x}=\ln|x|+C$:

$$I=\frac{1}{5}\ln|u|+C$$

www.cuet.pw www.cbse.page

Step 4: Back-Substitution

- Substitute the original value of $u=e^{5x}+1$ back into the equation.
- Note: Since $e^{5x}+1$ is always positive for all real x, we can remove the absolute value bars if we wish (though keeping them is never wrong).

$$I=\frac{1}{5}\ln\left(e^{5x}+1\right)+C$$

www.cuet.pw www.cbse.page

Method 2: Short Trick (General Formula)

If you encounter objective questions (multiple choice) or need to check your answer quickly, you can use the following generalization.

- For any integral of the form:

$$\int \frac{dx}{1+e^{-ax}}$$

- The answer is always:

$$\frac{1}{a}\ln\left(1+e^{ax}\right)+C$$

www.cuet.pw www.cbse.page

Verification using the Trick

- In our given problem: $\int \frac{dx}{1+e^{-5x}}$

- Here, the constant $a = 5$.
- Plugging $a = 5$ directly into the formula:

$$\text{Answer} = \frac{1}{5}\ln\left(1 + e^{5x}\right) + C$$

www.cuet.pw www.cbse.page

Final Answer

$$\int \frac{dx}{1 + e^{-5x}} = \frac{1}{5}\ln\left(1 + e^{5x}\right) + C$$

In terms of log

- Let the given integral be I.

$$I = \int \frac{dx}{1 + e^{-5x}}$$

- To simplify the integrand, multiply the numerator and the denominator by e^{5x}.

$$I = \int \frac{e^{5x} \cdot dx}{e^{5x}\left(1 + e^{-5x}\right)}$$

$$I = \int \frac{e^{5x}}{e^{5x} + e^{5x} \cdot e^{-5x}} dx$$

$$I = \int \frac{e^{5x}}{e^{5x} + 1} dx$$

- Now, we use the method of integration by substitution. Let $1 + e^{5x} = t$.
- Differentiating both sides with respect to x:

$$\frac{d}{dx}\left(1 + e^{5x}\right) = \frac{dt}{dx}$$

$$0 + 5e^{5x} = \frac{dt}{dx}$$

$$e^{5x} dx = \frac{dt}{5}$$

- Substitute these values back into the integral expression:

$$I = \int \frac{1}{t} \cdot \frac{dt}{5}$$

$$I = \frac{1}{5}\int \frac{1}{t} dt$$

- We know the standard integral formula $\int \frac{1}{x} dx = \log_e |x| + C$.

$$I = \frac{1}{5} \log_e |t| + C$$

- Substitute the value of $t = 1 + e^{5x}$ back into the equation.

$$I = \frac{1}{5} \log_e \left|1 + e^{5x}\right| + C$$

- Comparing this result with the given options, the correct answer is option (d).

CUET 2024 PYQ 12

The value of $\int_{-1}^{1} \left|\tan^{-1} x\right| dx$ is :

www.cuet.pw www.cbse.page

(1) $\frac{\pi}{2} - \log_e 2$

(2) $\frac{\pi}{2} + \log_e 2$

(3) $\frac{\pi - 1 - \log_e 2}{2}$

(4) $\frac{\pi - 1 + \log_e 2}{2}$

Concept Builder

www.cuet.pw www.cbse.page

Definite Integration of Modulus Functions

www.cuet.pw www.cbse.page

Problem Statement

Evaluate the integral:

$$I = \int_{-1}^{1} \left|\tan^{-1} x\right| dx$$

www.cuet.pw www.cbse.page

Step 1: Analyzing Symmetry (Even/Odd Properties)

- Let $f(x) = \tan^{-1} x$. We know that $\tan^{-1}(-x) = -\tan^{-1} x$ (Odd Function).
- However, the integrand is the **absolute value**: $g(x) = |\tan^{-1} x|$.
- Check $g(-x)$:

$$g(-x) = |\tan^{-1}(-x)| = |-\tan^{-1} x| = |\tan^{-1} x| = g(x)$$

- Since $g(-x) = g(x)$, the integrand is an **Even Function**.
- **Property:** For an even function, $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.

$$I = 2\int_{0}^{1} \tan^{-1} x \, dx$$

(Note: For $x \geq 0$, $\tan^{-1} x \geq 0$, so we can drop the modulus sign).

www.cuet.pw www.cbse.page

Step 2: Integration by Parts

- We need to integrate $\int \tan^{-1} x \, dx$. We use the method of parts where we treat 1 as the second function.
- **Formula:** $\int uv' dx = uv - \int u'v dx$.
- Let $u = \tan^{-1} x$ and $dv = 1 \, dx$.
- Then $du = \dfrac{1}{1+x^2} dx$ and $v = x$.
- Applying the formula:

$$\int \tan^{-1} x \, dx = x \tan^{-1} x - \int \frac{x}{1+x^2} dx$$

www.cuet.pw www.cbse.page

Step 3: Solve the second integral

- For $\int \dfrac{x}{1+x^2} dx$, use substitution.
- Let $1 + x^2 = t \implies 2x \, dx = dt \implies x \, dx = \dfrac{dt}{2}$.

$$\int \frac{1}{t} \cdot \frac{dt}{2} = \frac{1}{2} \ln |t| = \frac{1}{2} \ln \left(1 + x^2\right)$$

- So, the indefinite integral is:

$$\int \tan^{-1} x \, dx = x \tan^{-1} x - \frac{1}{2} \ln \left(1 + x^2\right)$$

www.cuet.pw www.cbse.page

Step 4: Apply Limits

- Recall from Step 1 that $I = 2 \left[x \tan^{-1} x - \dfrac{1}{2} \ln \left(1 + x^2\right) \right]_0^1$.
- **Upper Limit** ($x = 1$)**:**

$$\left(1 \cdot \tan^{-1}(1) - \frac{1}{2} \ln \left(1 + 1^2\right)\right) = \left(\frac{\pi}{4} - \frac{1}{2} \ln 2\right)$$

- **Lower Limit** ($x = 0$)**:**

$$\left(0 \cdot \tan^{-1}(0) - \frac{1}{2} \ln (1)\right) = (0 - 0) = 0$$

- **Combine:**

$$I = 2 \left[\frac{\pi}{4} - \frac{1}{2} \ln 2 \right]$$

- Distribute the 2:

$$I = \frac{\pi}{2} - \ln 2$$

- (Note: $\ln 2$ is the same as $\log_e 2$).

Correct Option: (1) $\frac{\pi}{2} - \log_e 2$

www.cuet.pw **www.cbse.page**

Short Trick / Visual Method (Area)

- **Graphing Inverse Functions:** Instead of integrating $y = \tan^{-1} x$ with respect to x, look at the area from the y-axis perspective.
- If $y = \tan^{-1} x$, then $x = \tan y$.
- The area under $y = \tan^{-1} x$ from $x = 0$ to $x = 1$ is inside a rectangle of width 1 and height $\pi/4$.
- Area = (Area of Rectangle) - (Area under $x = \tan y$ from $y = 0$ to $\pi/4$).

$$\int_0^1 \tan^{-1} x\,dx = (1 \cdot \frac{\pi}{4}) - \int_0^{\pi/4} \tan y\,dy$$

$$= \frac{\pi}{4} - [\ln|\sec y|]_0^{\pi/4}$$

$$= \frac{\pi}{4} - (\ln\sqrt{2} - \ln 1)$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln 2$$

- Multiply by 2 (for symmetry): $2\left(\frac{\pi}{4} - \frac{1}{2}\ln 2\right) = \frac{\pi}{2} - \ln 2$.
- This avoids the "Integration by Parts" formula entirely if you know the integral of $\tan y$.

CUET 2024 PYQ 13

For $f(x) = \int \frac{e^x}{\sqrt{4 - e^{2x}}} dx$, if the point $\left(0, \frac{\pi}{2}\right)$ satisfies $y = f(x)$, then the constant of integration of the given integral is :

www.cuet.pw www.cbse.page

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) 0

Concept Builder

www.cuet.pw www.cbse.page

Indefinite Integration with Initial Value Problem

www.cuet.pw www.cbse.page

Problem Statement

We are given the function defined by the integral:

$$f(x) = \int \frac{e^x}{\sqrt{4 - e^{2x}}} dx$$

We are also given a condition: The point $\left(0, \frac{\pi}{2}\right)$ lies on the curve $y = f(x)$.

We need to find the specific value of the Constant of Integration (C).

www.cuet.pw www.cbse.page

Step 1: Simplify the Integral using Substitution

- **Observation:** The integrand contains e^{2x}, which is $(e^x)^2$. The derivative of e^x is e^x, which is present in the numerator.
- **Substitution:** Let $u = e^x$. Differentiating with respect to x:

$$du = e^x dx$$

- **Rewrite the Integral:** Substitute u and du into the equation:

$$f(x) = \int \frac{1}{\sqrt{4 - u^2}} du$$

Rewrite 4 as 2^2:

$$f(x) = \int \frac{1}{\sqrt{2^2 - u^2}} du$$

www.cuet.pw www.cbse.page

Step 2: Apply Standard Integration Formula

- **Formula:** Recall the standard integral for inverse sine:

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

- **Application:** Here, variable is u and constant $a = 2$.

$$f(x) = \sin^{-1}\left(\frac{u}{2}\right) + C$$

- **Back-Substitution:** Replace u with e^x.

$$f(x) = \sin^{-1}\left(\frac{e^x}{2}\right) + C \quad \ldots \text{(General Solution)}$$

www.cuet.pw www.cbse.page

Step 3: Find the Constant 'C'

- We are given that the curve passes through $\left(0, \frac{\pi}{2}\right)$.
- This means when $x = 0$, $f(x) = \frac{\pi}{2}$.
- Substitute these values into the General Solution:

$$\frac{\pi}{2} = \sin^{-1}\left(\frac{e^0}{2}\right) + C$$

- Since $e^0 = 1$:

$$\frac{\pi}{2} = \sin^{-1}\left(\frac{1}{2}\right) + C$$

- Calculate the inverse sine value: We know that $\sin(\pi/6) = 1/2$ (or $\sin 30^\circ = 0.5$).

$$\frac{\pi}{2} = \frac{\pi}{6} + C$$

- Solve for C:

$$C = \frac{\pi}{2} - \frac{\pi}{6}$$

$$C = \frac{3\pi}{6} - \frac{\pi}{6}$$

$$C = \frac{2\pi}{6}$$

$$C = \frac{\pi}{3}$$

www.cuet.pw www.cbse.page

Final Answer

The constant of integration is $\frac{\pi}{3}$.

Correct Option: (b) $\frac{\pi}{3}$

www.cuet.pw www.cbse.page

Short Trick / Mental Flow

- **Step A: Recognize Form.** $\frac{f'(x)}{\sqrt{a^2 - f(x)^2}}$ is always $\sin^{-1}\left(\frac{f(x)}{a}\right)$. Here $f(x) = e^x, a = 2$. So immediately write: $\sin^{-1}\left(\frac{e^x}{2}\right) + C$.
- **Step B: Plug and Solve.** Input $x = 0 \rightarrow e^x = 1 \rightarrow \sin^{-1}(1/2) \rightarrow 30^\circ(\pi/6)$. Target is $90^\circ(\pi/2)$. Equation: $30^\circ + C = 90^\circ \implies C = 60^\circ(\pi/3)$.
- Working in degrees mentally is often faster for subtraction than fractions of π.

CUET 2024 PYQ 14

The value of $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\tan^{18} x}$ is :

www.cuet.pw www.cbse.page

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{9}$ (d) $\frac{\pi}{12}$

Concept Builder

www.cuet.pw www.cbse.page

Definite Integration: The King's Property

www.cuet.pw www.cbse.page

Problem Statement

Evaluate the integral:

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\tan^{18} x}$$

www.cuet.pw www.cbse.page

Concept: King's Property

- **Property:** $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- This property is extremely powerful when dealing with trigonometric functions over complementary limits.
- **Limits Check:** Here, $a = \frac{\pi}{6}$ and $b = \frac{\pi}{3}$.

$$a+b = \frac{\pi}{6} + \frac{\pi}{3} = 30^\circ + 60^\circ = 90^\circ = \frac{\pi}{2}$$

- So, we replace x with $\left(\frac{\pi}{2} - x\right)$.

www.cuet.pw www.cbse.page

Step-by-Step Solution

Step 1: Simplify the Original Integral

It is often easier to work with sines and cosines.

$$\tan^{18} x = \frac{\sin^{18} x}{\cos^{18} x}$$

Substitute this into the integral:

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1 + \frac{\sin^{18} x}{\cos^{18} x}} dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\cos^{18} x}{\cos^{18} x + \sin^{18} x} dx \quad \ldots \text{(Equation 1)}$$

Step 2: Apply King's Property

Replace $x \to \left(\frac{\pi}{2} - x\right)$.

$$I = \int_{\pi/6}^{\pi/3} \frac{\cos^{18}(\frac{\pi}{2} - x)}{\cos^{18}(\frac{\pi}{2} - x) + \sin^{18}(\frac{\pi}{2} - x)} dx$$

Recall that $\cos(90° - x) = \sin x$ and $\sin(90° - x) = \cos x$.

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{18} x}{\sin^{18} x + \cos^{18} x} dx \quad \ldots \text{(Equation 2)}$$

Step 3: Add Equation 1 and Equation 2

$$2I = \int_{\pi/6}^{\pi/3} \left(\frac{\cos^{18} x}{\cos^{18} x + \sin^{18} x} + \frac{\sin^{18} x}{\sin^{18} x + \cos^{18} x} \right) dx$$

Since the denominators are the same, we can add the numerators directly.

$$2I = \int_{\pi/6}^{\pi/3} \left(\frac{\cos^{18} x + \sin^{18} x}{\cos^{18} x + \sin^{18} x} \right) dx$$

The numerator and denominator cancel out perfectly!

$$2I = \int_{\pi/6}^{\pi/3} 1 \, dx$$

Step 4: Evaluate the Simple Integral

$$2I = [x]_{\pi/6}^{\pi/3}$$

$$2I = \left(\frac{\pi}{3} - \frac{\pi}{6} \right)$$

$$2I = \left(\frac{2\pi - \pi}{6} \right) = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}$$

Correct Option: (d) $\frac{\pi}{12}$

Short Trick / General Formula

- **Pattern Recognition:** For any integral of the form:

$$\int_a^b \frac{1}{1+\tan^n x}dx \quad \text{or} \quad \int_a^b \frac{1}{1+\cot^n x}dx$$

where $a+b=\frac{\pi}{2}$ (90 degrees), the answer is always:

$$\text{Value} = \frac{b-a}{2}$$

- **Why?** Because applying King's Property and adding always results in $\int 1\,dx$.
- **Application:**

$$\text{Upper Limit } (b) = \frac{\pi}{3}$$

$$\text{Lower Limit } (a) = \frac{\pi}{6}$$

$$\text{Answer} = \frac{\frac{\pi}{3}-\frac{\pi}{6}}{2} = \frac{\frac{\pi}{6}}{2} = \frac{\pi}{12}$$

- You can solve this in 5 seconds without writing any integration steps.

CUET 2024 PYQ 15

The value of $I = \int_0^{1.5} [x^2]\,dx$, where [ ] denotes the greatest integer function, is :

www.cuet.pw www.cbse.page

(a) $2 - \sqrt{2}$ (b) $\sqrt{2}$ (c) $5\sqrt{2}$ (d) $3 - 2\sqrt{2}$

Concept Builder

www.cuet.pw www.cbse.page

Definite Integration of Greatest Integer Function (GIF)

www.cuet.pw www.cbse.page

Problem Statement

Evaluate the integral:

$$I = \int_0^{1.5} [x^2]\,dx$$

where $[\cdot]$ denotes the greatest integer function.

www.cuet.pw www.cbse.page

Concept: Discontinuities in GIF

- The function $[t]$ is discontinuous whenever t is an integer.
- In our problem, the function is $[x^2]$.
- This function will change its value (jump) whenever x^2 becomes an integer $(0, 1, 2, 3 \ldots)$.
- We must identify these "critical points" to split the definite integral into continuous intervals.

www.cuet.pw www.cbse.page

Step 1: Determine Critical Points

- The range of integration is $x \in [0, 1.5]$.
- Let's check the range of the inner term x^2:

$$\text{Lower Limit: } x = 0 \Rightarrow x^2 = 0$$

$$\text{Upper Limit: } x = 1.5 \Rightarrow x^2 = (1.5)^2 = 2.25$$

- The integers falling between 0 and 2.25 are: **1 and 2**.
- We must split the integral at the x-values corresponding to these integers:

  (1) When $x^2 = 1 \Rightarrow x = 1$

  (2) When $x^2 = 2 \Rightarrow x = \sqrt{2}$ (approx 1.414, which is < 1.5)
- The splitting points for x are: $0 \to 1 \to \sqrt{2} \to 1.5$.

www.cuet.pw www.cbse.page

Step 2: Splitting the Integral

We divide the integral I into three parts:

$$I = \int_0^1 [x^2]\,dx + \int_1^{\sqrt{2}} [x^2]\,dx + \int_{\sqrt{2}}^{1.5} [x^2]\,dx$$

www.cuet.pw www.cbse.page

Step 3: Evaluating Each Interval

- **Interval 1:** $0 \le x < 1$

Here, $0 \le x^2 < 1$

Value of $[x^2] = 0$

$$I_1 = \int_0^1 0\,dx = 0$$

- **Interval 2:** $1 \le x < \sqrt{2}$

Here, $1 \le x^2 < 2$

Value of $[x^2] = 1$

$$I_2 = \int_1^{\sqrt{2}} 1\,dx = [x]_1^{\sqrt{2}} = (\sqrt{2} - 1)$$

- **Interval 3:** $\sqrt{2} \le x < 1.5$

Here, $2 \le x^2 \le 2.25$

Value of $[x^2] = 2$

$$I_3 = \int_{\sqrt{2}}^{1.5} 2\,dx = 2[x]_{\sqrt{2}}^{1.5}$$

$$I_3 = 2\left(1.5 - \sqrt{2}\right) = 3 - 2\sqrt{2}$$

www.cuet.pw www.cbse.page

Step 4: Final Summation

- Add the results of the three intervals:

$$I = I_1 + I_2 + I_3$$

$$I = 0 + (\sqrt{2} - 1) + (3 - 2\sqrt{2})$$

- Combine like terms:

$$I = (3 - 1) + (\sqrt{2} - 2\sqrt{2})$$

$$I = 2 - \sqrt{2}$$

Correct Option: (a) $2 - \sqrt{2}$

www.cuet.pw www.cbse.page

Short Trick / Graphical Method

[Image of Greatest Integer Graph Area]

- **Rectangle Concept:** The integral of a floor function $[f(x)]$ is essentially the sum of areas of rectangles.
- **Formula:** Area = (Height) × (Width)
- **Rect 1:** Height 0 (from $x = 0$ to 1). Area = 0.
- **Rect 2:** Height 1 (from $x = 1$ to $\sqrt{2}$).

$$\text{Area} = 1 \times (\sqrt{2} - 1) = \sqrt{2} - 1$$

- **Rect 3:** Height 2 (from $x = \sqrt{2}$ to 1.5).

$$\text{Area} = 2 \times (1.5 - \sqrt{2}) = 3 - 2\sqrt{2}$$

- **Total:** $(\sqrt{2} - 1) + (3 - 2\sqrt{2}) = 2 - \sqrt{2}$.
- *Why this is faster:* You skip writing the integration symbols and just calculate "Value × Interval Length" directly.

CUET 2024 PYQ 16

The integral of the function $\frac{1}{9-4x^2}$ is :

www.cuet.pw www.cbse.page

(a) $\frac{1}{22}\log_e\left|\frac{3+x}{3-x}\right|+C$, where C is an arbitrary constant

(b) $\frac{1}{12}\log_e\left|\frac{3+2x}{3-2x}\right|+C$, where C is an arbitrary constant

(c) $\frac{1}{2}\log_e\left|\frac{7+x}{7-x}\right|+C$, where C is an arbitrary constant

(d) $\frac{1}{12}\log_e\left|\frac{3-2x}{3+2x}\right|+C$, where C is an arbitrary constant

Concept Builder

www.cuet.pw www.cbse.page

Integration of Rational Functions

www.cuet.pw www.cbse.page

Problem Statement

We need to evaluate the integral:

$$I=\int\frac{1}{9-4x^2}dx$$

www.cuet.pw www.cbse.page

Method 1: Standard Formula Application

- **Standard Form:** The integral resembles the standard form:

$$\int\frac{1}{a^2-u^2}du=\frac{1}{2a}\log_e\left|\frac{a+u}{a-u}\right|+C$$

- **Transforming the Integral:** First, rewrite the denominator $9-4x^2$ to match the form a^2-u^2.

$$9-4x^2=(3)^2-(2x)^2$$

So, $a=3$ and our variable part is $(2x)$.

- **Substitution:** Let $u = 2x$. Differentiating with respect to x:

$$du = 2dx \implies dx = \frac{du}{2}$$

- **Substitute back into the Integral:**

$$I = \int \frac{1}{3^2 - u^2}\left(\frac{du}{2}\right)$$

$$I = \frac{1}{2}\int \frac{1}{3^2 - u^2}du$$

- **Apply the Formula:** Using $a = 3$:

$$I = \frac{1}{2}\left[\frac{1}{2(3)}\log_e\left|\frac{3+u}{3-u}\right|\right] + C$$

$$I = \frac{1}{2}\cdot\frac{1}{6}\log_e\left|\frac{3+u}{3-u}\right| + C$$

$$I = \frac{1}{12}\log_e\left|\frac{3+u}{3-u}\right| + C$$

- **Final Step:** Replace u with $2x$.

$$I = \frac{1}{12}\log_e\left|\frac{3+2x}{3-2x}\right| + C$$

www.cuet.pw www.cbse.page

Method 2: Partial Fractions (Conceptual Approach)

- Factor the denominator using $a^2 - b^2 = (a-b)(a+b)$.

$$\frac{1}{9-4x^2} = \frac{1}{(3-2x)(3+2x)}$$

- Split into partial fractions:

$$\frac{1}{(3-2x)(3+2x)} = \frac{A}{3-2x} + \frac{B}{3+2x}$$

$$1 = A(3+2x) + B(3-2x)$$

- **Find A:** Let $x = 3/2$ (makes B term zero).

$$1 = A(3+3) \implies 6A = 1 \implies A = 1/6$$

- **Find B:** Let $x = -3/2$ (makes A term zero).

$$1 = B(3-(-3)) \implies 6B = 1 \implies B = 1/6$$

- **Integrate:**

$$I = \int \left(\frac{1/6}{3-2x} + \frac{1/6}{3+2x} \right) dx$$

$$I = \frac{1}{6} \left[\frac{\ln|3-2x|}{-2} + \frac{\ln|3+2x|}{2} \right]$$

(Note the division by derivative of inside term)

$$I = \frac{1}{12} [\ln|3+2x| - \ln|3-2x|]$$

$$I = \frac{1}{12} \log_e \left| \frac{3+2x}{3-2x} \right| + C$$

www.cuet.pw www.cbse.page

Correct Option

Comparing with the given options, the answer matches: (b)

$\frac{1}{12} \log_e \left| \frac{3+2x}{3-2x} \right| + C$

www.cuet.pw www.cbse.page

Short Trick / Coefficient Rule

- **The Formula Adjustment:** Whenever you use a standard integral formula like $\int f(x)dx = F(x)$, if the variable x is replaced by a linear term $(mx+c)$, you must divide the result by the coefficient m.

$$\int f(mx+c)dx = \frac{1}{m} F(mx+c)$$

- **Direct Application:**

(1) Identify form $\frac{1}{a^2-x^2}$ which gives $\frac{1}{2a} \ln \ldots$.

(2) Here $a = 3$, so term is $\frac{1}{2(3)} = \frac{1}{6}$.

(3) The variable is $2x$, so coefficient $m = 2$.

(4) Divide the front term by 2: $\frac{1}{6} \nabla \cdot 2 = \frac{1}{12}$.

(5) Write the log term directly: $\log \left| \frac{3+2x}{3-2x} \right|$.

This gets you the answer in 10 seconds without substitution.

CUET 2024 PYQ 17

The value of $\int_0^1 \frac{a-bx^2}{\left(a+bx^2\right)^2}dx$ is :

www.cuet.pw www.cbse.page

(1) $\frac{a-b}{a+b}$ (2) $\frac{1}{a-b}$ (3) $\frac{a+b}{2}$ (4) $\frac{1}{a+b}$

Concept Builder

www.cuet.pw www.cbse.page

- **Integration as Reverse Differentiation:** If we can recognize that the integrand $f(x)$ is the derivative of some function $g(x)$, then $\int f(x)dx = g(x)$.

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2} \quad \text{(Quotient Rule)}$$

- **Trigonometric Substitution:** A standard method for integrals involving terms like $a+bx^2$. We substitute x with a trigonometric function (usually $\tan\theta$) to simplify the expression using identities like $1+\tan^2\theta = \sec^2\theta$.

www.cuet.pw www.cbse.page

Method 1: Inspection / Reverse Differentiation (Short Trick)

- **Concept:** Look closely at the structure of the integrand $\frac{a-bx^2}{\left(a+bx^2\right)^2}$. The denominator is squared, which strongly suggests the Quotient Rule of differentiation was applied to a function with denominator $(a+bx^2)$.
- **Step 1: Test the derivative of** $\frac{x}{a+bx^2}$ Let $y = \frac{x}{a+bx^2}$. Apply the Quotient Rule:

$$\frac{dy}{dx} = \frac{\left(a+bx^2\right)\cdot\frac{d}{dx}(x) - x\cdot\frac{d}{dx}\left(a+bx^2\right)}{\left(a+bx^2\right)^2}$$

$$\frac{dy}{dx} = \frac{\left(a+bx^2\right)(1) - x(2bx)}{\left(a+bx^2\right)^2}$$

$$\frac{dy}{dx} = \frac{a+bx^2-2bx^2}{\left(a+bx^2\right)^2}$$

$$\frac{dy}{dx} = \frac{a-bx^2}{\left(a+bx^2\right)^2}$$

- **Step 2: Solve the Integral directly** Since the integrand is exactly the derivative of $\frac{x}{a+bx^2}$, the integral is simply the function itself.

$$I = \int_0^1 \frac{a-bx^2}{\left(a+bx^2\right)^2} dx = \left[\frac{x}{a+bx^2}\right]_0^1$$

- **Step 3: Apply Limits** Upper Limit ($x = 1$):

$$\frac{1}{a+b(1)^2} = \frac{1}{a+b}$$

Lower Limit ($x = 0$):

$$\frac{0}{a+b(0)^2} = 0$$

Result:

$$I = \frac{1}{a+b} - 0 = \frac{1}{a+b}$$

www.cuet.pw www.cbse.page

Method 2: Trigonometric Substitution (Standard Method)

- **Step 1: Substitution** Let $x = \sqrt{\frac{a}{b}}\tan\theta$. Then $dx = \sqrt{\frac{a}{b}}\sec^2\theta\, d\theta$.

Also, the term $a+bx^2$ becomes:

$$a + b\left(\frac{a}{b}\tan^2\theta\right) = a(1+\tan^2\theta) = a\sec^2\theta$$

And the numerator $a-bx^2$ becomes:

$$a - b\left(\frac{a}{b}\tan^2\theta\right) = a(1-\tan^2\theta)$$

- **Step 2: Change Limits**

✓ When $x = 0$, $\tan\theta = 0 \implies \theta = 0$.

When $x = 1$, $\sqrt{\frac{a}{b}}\tan\theta = 1 \implies \tan\theta = \sqrt{\frac{b}{a}} \implies \theta = \tan^{-1}\left(\sqrt{\frac{b}{a}}\right)$.

Let $\alpha = \tan^{-1}\left(\sqrt{\frac{b}{a}}\right)$.

- **Step 3: Simplify Integral**

$$I = \int_0^{\alpha} \frac{a(1-\tan^2\theta)}{(a\sec^2\theta)^2} \cdot \sqrt{\frac{a}{b}}\sec^2\theta\, d\theta$$

$$I = \sqrt{\frac{a}{b}}\int_0^{\alpha} \frac{a(1-\tan^2\theta)}{a^2\sec^4\theta} \cdot \sec^2\theta\, d\theta$$

$$I = \frac{1}{\sqrt{ab}}\int_0^{\alpha} \frac{1-\tan^2\theta}{\sec^2\theta}\, d\theta$$

Using trig identities ($1-\tan^2\theta = \frac{\cos 2\theta}{\cos^2\theta}$ and $\sec^2\theta = \frac{1}{\cos^2\theta}$):

$$\frac{1-\tan^2\theta}{\sec^2\theta} = \cos^2\theta - \sin^2\theta = \cos 2\theta$$

So,

$$I = \frac{1}{\sqrt{ab}}\int_0^{\alpha} \cos 2\theta\, d\theta$$

- **Step 4: Integration**

$$I = \frac{1}{\sqrt{ab}}\left[\frac{\sin 2\theta}{2}\right]_0^{\alpha}$$

$$I = \frac{1}{2\sqrt{ab}}\sin 2\alpha$$

Expanding $\sin 2\alpha = 2\sin\alpha\cos\alpha$:

$$I = \frac{1}{\sqrt{ab}}\sin\alpha\cos\alpha$$

From $\tan\alpha = \sqrt{\frac{b}{a}}$, we can form a triangle with opposite $\sqrt{b}$ and adjacent $\sqrt{a}$. The hypotenuse is $\sqrt{a+b}$.

$$\sin\alpha = \frac{\sqrt{b}}{\sqrt{a+b}}, \quad \cos\alpha = \frac{\sqrt{a}}{\sqrt{a+b}}$$

$$I = \frac{1}{\sqrt{ab}} \cdot \frac{\sqrt{b}}{\sqrt{a+b}} \cdot \frac{\sqrt{a}}{\sqrt{a+b}}$$

$$I = \frac{\sqrt{ab}}{\sqrt{ab}(a+b)} = \frac{1}{a+b}$$

www.cuet.pw www.cbse.page

Final Answer

- The value of the integral is $\frac{1}{a+b}$.
- Correct Option: **(4)**

CUET 2024 PYQ 18

$$\int e^x \left(\frac{2x+1}{2\sqrt{x}}\right) dx =$$

www.cuet.pw www.cbse.page

(1) $\frac{1}{2\sqrt{x}}e^x + C$ (2) $-e^x\sqrt{x} + C$ (3) $-\frac{1}{2\sqrt{x}}e^x + C$ (4) $e^x\sqrt{x} + C$

Concept Builder

www.cuet.pw www.cbse.page

- **Standard Integration Property:** The most powerful concept for integrals involving e^x is:

$$\int e^x \left[f(x) + f'(x)\right] dx = e^x f(x) + C$$

This means if you can split the term inside the bracket into a function and its derivative, the answer is simply e^x times the function.

- **Differentiation Power Rule:**

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

www.cuet.pw www.cbse.page

Method 1: Splitting into $f(x) + f'(x)$ (Standard Method)

- **Step 1: Simplify the algebraic term**

The given integral is:

$$I = \int e^x \left(\frac{2x+1}{2\sqrt{x}}\right) dx$$

Separate the fraction inside the bracket:

$$\frac{2x+1}{2\sqrt{x}} = \frac{2x}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}$$

Simplify the first term ($\frac{x}{\sqrt{x}} = \sqrt{x}$):

$$= \sqrt{x} + \frac{1}{2\sqrt{x}}$$

- **Step 2: Identify $f(x)$ and $f'(x)$**

  Let $f(x) = \sqrt{x} = x^{1/2}$. Now find its derivative $f'(x)$:

$$f'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

  Notice that the second term we derived in Step 1 is exactly this derivative!

$$\text{Expression} = e^x \left[\underbrace{\sqrt{x}}_{f(x)} + \underbrace{\frac{1}{2\sqrt{x}}}_{f'(x)} \right]$$

- **Step 3: Apply the Formula**

  Since the integral matches the form $\int e^x [f(x) + f'(x)]\,dx$:

$$I = e^x f(x) + C$$

$$I = e^x \sqrt{x} + C$$

www.cuet.pw www.cbse.page

Method 2: Reverse Differentiation (Short Trick)

- **Concept:** Since integration is the reverse of differentiation, you can differentiate the options to see which one gives back the integrand.

  The integrand is $e^x \left(\frac{2x+1}{2\sqrt{x}} \right)$.

- **Test Option (4):** $y = e^x \sqrt{x}$ Apply Product Rule $(uv' + vu')$:

$$\frac{dy}{dx} = e^x \left(\frac{d}{dx}(\sqrt{x}) \right) + \sqrt{x} \left(\frac{d}{dx}(e^x) \right)$$

$$\frac{dy}{dx} = e^x \left(\frac{1}{2\sqrt{x}} \right) + \sqrt{x}(e^x)$$

Factor out e^x:

$$\frac{dy}{dx} = e^x \left(\frac{1}{2\sqrt{x}} + \sqrt{x} \right)$$

Take common denominator $(2\sqrt{x})$ inside the bracket:

$$\frac{dy}{dx} = e^x \left(\frac{1 + 2(\sqrt{x})(\sqrt{x})}{2\sqrt{x}} \right)$$

$$\frac{dy}{dx} = e^x \left(\frac{1+2x}{2\sqrt{x}} \right)$$

This matches the question exactly.

www.cuet.pw www.cbse.page

Final Answer

- The value of the integral is $e^x \sqrt{x} + C$.
- Correct Option: **(4)**

CUET 2024 PYQ 19

$$\int_0^{\frac{\pi}{2}} \frac{1-\cot x}{\operatorname{cosec} x+\cos x}dx =$$

www.cuet.pw www.cbse.page

(1) 0 (2) $\frac{\pi}{4}$ (3) ∞ (4) $\frac{\pi}{12}$

Concept Builder

www.cuet.pw www.cbse.page

- **Trigonometric Identities:**

$$\cot x = \frac{\cos x}{\sin x}, \quad \operatorname{cosec} x = \frac{1}{\sin x}$$

- **King's Property of Definite Integration:**

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

This is extremely useful for simplifying integrals involving trigonometric functions over $[0, \pi/2]$.

- **Complementary Angles:**

$$\sin(\pi/2 - x) = \cos x, \quad \cos(\pi/2 - x) = \sin x$$

www.cuet.pw www.cbse.page

Method 1: Simplification and King's Property (Standard Method)

- **Step 1: Simplify the Integrand**

Let $I = \int_0^{\frac{\pi}{2}} \frac{1-\cot x}{\operatorname{cosec} x+\cos x}dx.$

Convert everything to sine and cosine:

$$\text{Numerator} = 1 - \frac{\cos x}{\sin x} = \frac{\sin x - \cos x}{\sin x}$$

$$\text{Denominator} = \frac{1}{\sin x} + \cos x = \frac{1 + \sin x \cos x}{\sin x}$$

Now divide Numerator by Denominator:

$$\frac{\frac{\sin x-\cos x}{\sin x}}{\frac{1+\sin x\cos x}{\sin x}}=\frac{\sin x-\cos x}{1+\sin x\cos x}$$

So the simplified integral is:

$$I=\int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x}{1+\sin x\cos x}dx \quad \ldots\ldots\text{(Equation 1)}$$

- **Step 2: Apply King's Property**

Replace x with $(\frac{\pi}{2}-x)$:

$$I=\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx$$

Using complementary identities:

$$I=\int_0^{\frac{\pi}{2}}\frac{\cos x-\sin x}{1+\cos x\sin x}dx \quad \ldots\ldots\text{(Equation 2)}$$

- **Step 3: Add Equation 1 and Equation 2**

$$2I=\int_0^{\frac{\pi}{2}}\left(\frac{\sin x-\cos x}{1+\sin x\cos x}+\frac{\cos x-\sin x}{1+\sin x\cos x}\right)dx$$

Since the denominators are the same, just add the numerators:

$$2I=\int_0^{\frac{\pi}{2}}\frac{(\sin x-\cos x)+(\cos x-\sin x)}{1+\sin x\cos x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}\frac{0}{1+\sin x\cos x}dx$$

$$2I=0$$

$$I=0$$

www.cuet.pw www.cbse.page

Method 2: Short Trick (Symmetry Observation)

- **Concept:** Whenever you see an integrand of the form $\frac{\sin x-\cos x}{f(\sin x\cos x)}$ integrated from 0 to $\pi/2$, the answer is almost always zero.
- **Visual Logic:** The function $g(x)=\sin x-\cos x$ is antisymmetric about the midpoint $\pi/4$.

- For $x \in [0, \pi/4)$, cos > sin (Negative value).
- For $x \in (\pi/4, \pi/2]$, sin > cos (Positive value).

Since the denominator is symmetric ($1 + \sin x \cos x$ doesn't change if you swap sine and cosine), the positive area exactly cancels the negative area.

- **Direct Answer:** 0.

www.cuet.pw www.cbse.page

Final Answer

- The value of the integral is 0.
- Correct Option: **(1)**

CUET 2024 PYQ 20

The value of the integral $\int_{\log_e 2}^{\log_e 3} \frac{e^{2x}-1}{e^{2x}+1}dx$ is :

www.cuet.pw www.cbse.page

(1) $\log_e 3$ (2) $\log_e 4 - \log_e 3$ (3) $\log_e 9 - \log_e 4$ (4) $\log_e 3 - \log_e 2$

Concept Builder

www.cuet.pw www.cbse.page

- **Integration of $f'(x)/f(x)$:** If the numerator is the exact derivative of the denominator, the integral is the natural logarithm of the denominator.

$$\int \frac{f'(x)}{f(x)}dx = \log_e |f(x)| + C$$

- **Logarithm Properties:**

$$\log_e a - \log_e b = \log_e \left(\frac{a}{b}\right)$$

$$e^{\log_e x} = x \quad \text{and} \quad e^{-\log_e x} = e^{\log_e (1/x)} = \frac{1}{x}$$

www.cuet.pw www.cbse.page

Method 1: Reduction to $f'(x)/f(x)$ Form (Standard Method)

- **Step 1: Simplify the Integrand**

The given integral is $I = \int_{\log_e 2}^{\log_e 3} \frac{e^{2x}-1}{e^{2x}+1}dx.$

Divide both the numerator and the denominator by e^x to create a symmetric form:

$$\frac{e^{2x}-1}{e^{2x}+1} = \frac{\frac{e^{2x}}{e^x} - \frac{1}{e^x}}{\frac{e^{2x}}{e^x} + \frac{1}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

So the integral becomes:

$$I = \int_{\log_e 2}^{\log_e 3} \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$$

- **Step 2: Substitution**

  Let the denominator be t:

$$t = e^{x} + e^{-x}$$

  Differentiate with respect to x:

$$dt = \left(e^{x} - e^{-x}\right) dx$$

  Notice that this matches the numerator exactly.

- **Step 3: Change the Limits**

  - **Lower Limit:** When $x = \log_e 2$:

$$t = e^{\log_e 2} + e^{-\log_e 2} = 2 + \frac{1}{2} = \frac{5}{2}$$

  - **Upper Limit:** When $x = \log_e 3$:

$$t = e^{\log_e 3} + e^{-\log_e 3} = 3 + \frac{1}{3} = \frac{10}{3}$$

- **Step 4: Solve the Integral**

$$I = \int_{5/2}^{10/3} \frac{1}{t} dt$$

$$I = \left[\log_e t\right]_{5/2}^{10/3}$$

  Applying limits:

$$I = \log_e \left(\frac{10}{3}\right) - \log_e \left(\frac{5}{2}\right)$$

- **Step 5: Simplify using Log Properties**

$$I = \log_e \left(\frac{10/3}{5/2}\right)$$

$$I = \log_e \left(\frac{10}{3} \times \frac{2}{5}\right)$$

$$I = \log_e \left(\frac{4}{3}\right)$$

$$I = \log_e 4 - \log_e 3$$

www.cuet.pw www.cbse.page

Method 2: Short Trick (Hyperbolic Functions)

- **Concept:** Recognize that $\frac{e^{2x}-1}{e^{2x}+1} = \tanh x$. The integral of $\tanh x$ is standard: $\log_e|\cosh x|$.
- **Execution:**

$$\int \tanh x \, dx = \left[\log_e(e^x + e^{-x})\right]_{\text{limits}} - \log_e 2$$

(Since $\cosh x = \frac{e^x+e^{-x}}{2}$, but constants cancel in definite integrals, we effectively focus on $\log_e(e^x + e^{-x})$).

Evaluate $f(x) = \log_e(e^x + e^{-x})$ at limits:

- Upper ($x = \log 3$): $\log_e(3 + 1/3) = \log_e(10/3)$
- Lower ($x = \log 2$): $\log_e(2 + 1/2) = \log_e(5/2)$

Difference:

$$\log_e(10/3) - \log_e(5/2) = \log_e(4/3) = \log_e 4 - \log_e 3$$

www.cuet.pw www.cbse.page

Final Answer

- The value of the integral is $\log_e 4 - \log_e 3$.
- Correct Option: **(2)**

CUET 2024 PYQ 21

$$\int \frac{\pi}{x^{n+1}-x} dx =$$

www.cuet.pw www.cbse.page

(1) $\frac{\pi}{n}\log_e\left|\frac{x^n-1}{x^n}\right|+C$

(2) $\log_e\left|\frac{x^n+1}{x^n-1}\right|+C$

(3) $\frac{\pi}{n}\log_e\left|\frac{x^n+1}{x^n}\right|+C$

(4) $\pi\log_e\left|\frac{x^n}{x^n-1}\right|+C$

Concept Builder

www.cuet.pw www.cbse.page

- **Integration by Substitution:** A method to simplify an integral by changing the variable of integration. Ideally, we look for a function and its derivative present in the integrand.

$$\int f\left(g(x)\right)g'(x)dx = \int f(u)du$$

- **Partial Fractions:** Used to decompose a rational function into simpler rational functions that are easier to integrate.

$$\frac{1}{u(u-1)} = \frac{1}{u-1} - \frac{1}{u}$$

- **Logarithmic Integral:**

$$\int \frac{1}{x}dx = \log_e|x| + C$$

www.cuet.pw www.cbse.page

Method 1: Standard Substitution (Multiply and Divide)

- **Step 1: Simplify the Denominator**

  Given integral:

$$I = \int \frac{\pi}{x^{n+1} - x} dx$$

  Factor out x from the denominator:

$$I = \int \frac{\pi}{x\,(x^n - 1)} dx$$

- **Step 2: Create the Derivative**

  To make substitution easier, multiply the numerator and denominator by x^{n-1}. This creates x^n in the denominator and its derivative (proportional to x^{n-1}) in the numerator.

$$I = \int \frac{\pi \cdot x^{n-1}}{x \cdot x^{n-1}\,(x^n - 1)} dx$$

$$I = \pi \int \frac{x^{n-1}}{x^n\,(x^n - 1)} dx$$

- **Step 3: Apply Substitution**

  Let $u = x^n$. Differentiating with respect to x:

$$du = nx^{n-1} dx \implies x^{n-1} dx = \frac{du}{n}$$

  Substitute this into the integral:

$$I = \pi \int \frac{1}{u(u-1)} \cdot \frac{du}{n}$$

$$I = \frac{\pi}{n} \int \frac{1}{u(u-1)} du$$

- **Step 4: Use Partial Fractions**

  Decompose the fraction $\frac{1}{u(u-1)}$:

$$\frac{1}{u(u-1)} = \frac{1}{u-1} - \frac{1}{u}$$

  Now integrate:

$$I = \frac{\pi}{n} \int \left(\frac{1}{u-1} - \frac{1}{u} \right) du$$

$$I = \frac{\pi}{n} \left[\log_e |u-1| - \log_e |u| \right] + C$$

- **Step 5: Simplify and Resubstitute**

  Using log property $\log a - \log b = \log\left(\frac{a}{b}\right)$:

$$I = \frac{\pi}{n}\log_e\left|\frac{u-1}{u}\right| + C$$

  Substitute back $u = x^n$:

$$I = \frac{\pi}{n}\log_e\left|\frac{x^n-1}{x^n}\right| + C$$

www.cuet.pw www.cbse.page

Method 2: Short Trick (Forcing the Derivative)

- **Concept:** For integrals of the form $\int \frac{dx}{x(x^n \pm 1)}$ or $\int \frac{dx}{x^{n+1} \pm x}$, factor out the highest power of x from the denominator. This often leads immediately to a form $\int \frac{f'(x)}{f(x)}dx$.
- **Step 1: Factor out x^{n+1}**

$$I = \int \frac{\pi}{x^{n+1} - x}dx$$

  Factor x^{n+1} from the denominator:

$$I = \int \frac{\pi}{x^{n+1}\left(1 - \frac{x}{x^{n+1}}\right)}dx$$

$$I = \pi \int \frac{1}{x^{n+1}(1 - x^{-n})}dx$$

- **Step 2: Substitute the Bracket Term**

  Let $t = 1 - x^{-n}$. Differentiate:

$$dt = 0 - (-n)x^{-n-1}dx$$

$$dt = \frac{n}{x^{n+1}}dx \implies \frac{1}{x^{n+1}}dx = \frac{dt}{n}$$

- **Step 3: Integrate Directly**

$$I = \pi \int \frac{1}{t} \cdot \frac{dt}{n}$$

$$I = \frac{\pi}{n}\log_e|t| + C$$

- **Step 4: Final Substitution**

  Replace t:

$$I = \frac{\pi}{n} \log_e \left| 1 - \frac{1}{x^n} \right| + C$$

$$I = \frac{\pi}{n} \log_e \left| \frac{x^n - 1}{x^n} \right| + C$$

www.cuet.pw www.cbse.page

Final Answer

- The integral evaluates to $\frac{\pi}{n} \log_e \left| \frac{x^n - 1}{x^n} \right| + C$.
- Correct Option: **(1)**

NCET 2024 PYQ 22

If $f'(x) = 3x^2 - \frac{2}{x^3}$ and $f(1) = 0$, then $f(x)$ is equal to :

www.cuet.pw www.cbse.page

(a) $x^3 + \frac{1}{x^2} - 2$ (b) $x^3 + \frac{1}{x^2} + 2$ (c) $x^3 - \frac{1}{x^2} + 2$ (d) $x^3 - \frac{1}{x^4} - 2$

Concept Builder

1. Core Concepts

- **Antiderivative (Integration):** The process of finding the original function $f(x)$ from its derivative $f'(x)$ is called integration.

$$f(x) = \int f'(x)\,dx$$

- **Power Rule for Integration:**

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (\text{for } n \neq -1)$$

- **Arbitrary Constant (C):** Integration always produces an unknown constant C. We find the specific value of C using the given initial condition (in this case, $f(1) = 0$).

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

(1) **Step 1: Integrate the Derivative** Given $f'(x) = 3x^2 - \frac{2}{x^3}$. First, rewrite the term $\frac{2}{x^3}$ using negative exponents:

$$f'(x) = 3x^2 - 2x^{-3}$$

Now, integrate with respect to x:

$$f(x) = \int (3x^2 - 2x^{-3})\,dx$$

(2) **Step 2: Apply the Power Rule** Integrate each term separately:

- Term 1: $\int 3x^2\,dx = 3 \cdot \dfrac{x^{2+1}}{2+1} = 3 \cdot \dfrac{x^3}{3} = x^3$
- Term 2: $\int -2x^{-3}\,dx = -2 \cdot \dfrac{x^{-3+1}}{-3+1} = -2 \cdot \dfrac{x^{-2}}{-2} = x^{-2} = \dfrac{1}{x^2}$

So, the general solution is:

$$f(x) = x^3 + \frac{1}{x^2} + C$$

(3) **Step 3: Use the Initial Condition to Find** C We are given that $f(1) = 0$. Substitute $x = 1$ and $f(x) = 0$ into our general solution:

$$0 = (1)^3 + \frac{1}{(1)^2} + C$$

$$0 = 1 + 1 + C$$

$$0 = 2 + C$$

$$C = -2$$

(4) **Step 4: Write the Final Function** Substitute $C = -2$ back into the general solution:

$$f(x) = x^3 + \frac{1}{x^2} - 2$$

www.cuet.pw www.cbse.page

3. Short Tricks (Reverse Engineering)

- **Check the Derivative First:** Differentiate the options mentally.
  - Option (a) & (b) term $\dfrac{1}{x^2} = x^{-2} \xrightarrow{\frac{d}{dx}} -2x^{-3} = -\dfrac{2}{x^3}$. (Matches question).
  - Option (c) term $-\dfrac{1}{x^2} = -x^{-2} \xrightarrow{\frac{d}{dx}} +2x^{-3}$. (Wrong Sign).
  - Option (d) term $-\dfrac{1}{x^4}$. (Wrong Power).

  Only (a) and (b) are candidates.
- **Check the Initial Condition:** Plug $x = 1$ into the remaining candidates (a) and (b). The result must be 0.
  - (a) $1^3 + \frac{1}{1^2} - 2 = 1 + 1 - 2 = 0$. **(Correct)**
  - (b) $1^3 + \frac{1}{1^2} + 2 = 1 + 1 + 2 = 4$. **(Incorrect)**

www.cuet.pw www.cbse.page

4. Final Answer

- The function is $f(x) = x^3 + \dfrac{1}{x^2} - 2$.
- This corresponds to option **(a)**.

NCET 2024 PYQ 23

If $\int_0^1 \frac{(9^x+3^x)^2}{3^{3x}}\mathrm{d}x = \frac{a^b}{\log_e b^b} + a$, then $a+b$ is equal to :

www.cuet.pw www.cbse.page

(a) 3 (b) 5 (c) 7 (d) 9

Concept Builder

1. Core Concepts

- **Exponent Laws:** We use laws like $(a^m)^n = a^{mn}$ and $\frac{a^m}{a^n} = a^{m-n}$ to simplify the integrand before integrating.
- **Standard Integral:** The integration formula for exponential functions is:

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

- **Logarithm Properties:** To match the final answer with the given form, we use $\log_e(x^y) = y\log_e x$.

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

(1) Step 1: Simplify the Integrand Let $I = \int_0^1 \frac{(9^x+3^x)^2}{3^{3x}}\mathrm{d}x$.

First, express everything in base 3. Note that $9^x = (3^2)^x = 3^{2x}$.

$$\text{Numerator} = (3^{2x}+3^x)^2 = (3^{2x})^2 + (3^x)^2 + 2(3^{2x})(3^x)$$

$$= 3^{4x} + 3^{2x} + 2\cdot 3^{3x}$$

Now divide by the denominator 3^{3x}:

$$\frac{3^{4x}+3^{2x}+2\cdot 3^{3x}}{3^{3x}} = \frac{3^{4x}}{3^{3x}} + \frac{3^{2x}}{3^{3x}} + \frac{2\cdot 3^{3x}}{3^{3x}}$$

Using the quotient rule for exponents ($3^m/3^n = 3^{m-n}$):

$$= 3^{4x-3x} + 3^{2x-3x} + 2$$

$$= 3^x + 3^{-x} + 2$$

So, the integral simplifies to:

$$I = \int_0^1 (3^x + 3^{-x} + 2)\, dx$$

(2) Step 2: Evaluate the Integral We integrate each term separately:

$$I = \left[\frac{3^x}{\ln 3} + \frac{3^{-x}}{-\ln 3} + 2x \right]_0^1$$

$$I = \left[\frac{3^x}{\ln 3} - \frac{1}{3^x \ln 3} + 2x \right]_0^1$$

(3) Step 3: Apply Limits Upper Limit ($x = 1$):

$$\left(\frac{3^1}{\ln 3} - \frac{1}{3^1 \ln 3} + 2(1) \right) = \frac{3}{\ln 3} - \frac{1}{3\ln 3} + 2$$

$$= \frac{1}{\ln 3}\left(3 - \frac{1}{3}\right) + 2 = \frac{1}{\ln 3}\left(\frac{8}{3}\right) + 2 = \frac{8}{3\ln 3} + 2$$

Lower Limit ($x = 0$):

$$\left(\frac{3^0}{\ln 3} - \frac{1}{3^0 \ln 3} + 2(0) \right) = \frac{1}{\ln 3} - \frac{1}{\ln 3} + 0 = 0$$

So, the value of the integral is:

$$I = \frac{8}{3\ln 3} + 2$$

(4) Step 4: Compare with Given Form We are given $I = \dfrac{a^b}{\log_e b^b} + a$. Recall that $\log_e b^b = b \ln b$. So we compare:

$$\frac{8}{3\ln 3} + 2 \equiv \frac{a^b}{b \ln b} + a$$

Comparing the constant term:

$$a = 2$$

Now check the fraction with $a = 2$:

$$\frac{2^b}{b \ln b} = \frac{8}{3 \ln 3}$$

For the numerators to match: $2^b = 8 \implies 2^b = 2^3 \implies b = 3$. For the denominators to match: $b \ln b = 3 \ln 3$. This works perfectly.

So, $a = 2$ and $b = 3$.

(5) Step 5: Find $a + b$

$$a + b = 2 + 3 = 5$$

www.cuet.pw www.cbse.page

3. Short Tricks

- **Constant Matching:** In problems of the form "Expression = Value + Constant", usually the standalone constant term in your calculated result corresponds directly to the variable in that position. Here, the "+ 2" in our result immediately suggested $a = 2$.
- **Perfect Square Observation:** Notice $(9^x + 3^x)^2 / 3^{3x}$. $9^x/3^{1.5x} = 3^{2x}/3^{1.5x} = 3^{0.5x}\ 3^x/3^{1.5x} = 3^{-0.5x}$ Actually, simplified, the integrand is just $(3^{x/2} + 3^{-x/2})^2$. Expanding this gives $3^x + 2 + 3^{-x}$ instantly.

www.cuet.pw www.cbse.page

4. Final Answer

- The value of $a + b$ is 5.
- This corresponds to option **(b)**.

NCET 2024 PYQ 24

$\int_{-\pi}^{\pi} \frac{\sin x}{1+\sin^2 x} dx$ is equal to :

www.cuet.pw www.cbse.page

(a) π (b) 2π (c) π^2 (d) 0

Concept Builder

1. Core Concepts

- **Symmetric Interval Property:** When a definite integral has limits from $-a$ to a (a symmetric interval around the origin), we must check if the function is **Odd** or **Even**.
- **Odd Function:** A function $f(x)$ is odd if $f(-x) = -f(x)$. (Graph is symmetric about the origin).
- **Even Function:** A function $f(x)$ is even if $f(-x) = f(x)$. (Graph is symmetric about the y-axis).
- **The Golden Rule:**

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0 & \text{if } f(x) \text{ is Odd} \\ 2\int_0^a f(x)\,dx & \text{if } f(x) \text{ is Even} \end{cases}$$

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

(1) **Step 1: Identify Limits and Function** Limits are $-\pi$ to π (Symmetric).
Let $f(x) = \frac{\sin x}{1+\sin^2 x}$.

(2) **Step 2: Test for Parity (Odd/Even)** Replace x with $-x$ in the function:

$$f(-x) = \frac{\sin(-x)}{1+\sin^2(-x)}$$

Recall trigonometric properties:

- $\sin(-x) = -\sin x$

- $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$

Substitute these back:

$$f(-x) = \frac{-\sin x}{1+\sin^2 x}$$

$$f(-x) = -\left(\frac{\sin x}{1+\sin^2 x}\right)$$

$$f(-x) = -f(x)$$

(3) **Step 3: Conclusion** Since $f(-x) = -f(x)$, the function $f(x)$ is an **Odd Function**. Therefore, the integral over the symmetric interval is zero.

$$\int_{-\pi}^{\pi} \frac{\sin x}{1+\sin^2 x}\, dx = 0$$

www.cuet.pw www.cbse.page

3. Short Trick (The "Symmetry Check")

- **Reflex Action:** Whenever you see limits like $\int_{-a}^{a}$, **DO NOT** start integrating immediately.
- **Mental Check:** Look at the powers of x or the trig functions.
  - ✓ $\sin x$ is Odd.
  - ✓ $\sin^2 x$ is Even.
  - ✓ Odd/Even = Odd.
- Since the result is Odd, the answer is instantly **0**.

www.cuet.pw www.cbse.page

4. Final Answer

- The value of the integral is 0.
- This corresponds to option **(d)**.

NCET 2024 PYQ 25

The Integrating factor of the differential equation $(x+2y^2)\dfrac{dy}{dx}=y$, is :

www.cuet.pw www.cbse.page

(a) $\dfrac{1}{x}$ (b) $\dfrac{1}{y}$ (c) $-\dfrac{1}{x}$ (d) $-\dfrac{1}{y}$

Concept Builder

1. Core Concepts

- **Linear Differential Equation (Standard Form):** Usually, we look for the form $\dfrac{dy}{dx}+Py=Q$, where P and Q are functions of x.
- **Linear Differential Equation (Alternate Form):** Sometimes, the equation is not linear in y but is linear in x. The form is:

$$\frac{dx}{dy}+P_1x=Q_1$$

where P_1 and Q_1 are functions of y.

- **Integrating Factor (I.F.):** For the alternate form, the Integrating Factor is calculated as:

$$\text{I.F.}=e^{\int P_1\,dy}$$

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

(1) **Step 1: Analyze the Equation Structure** Given: $(x+2y^2)\dfrac{dy}{dx}=y$.

If we try to isolate $\dfrac{dy}{dx}$, we get $\dfrac{dy}{dx}=\dfrac{y}{x+2y^2}$. This is difficult to separate because the denominator has a sum.

Strategy: Flip the equation to find $\dfrac{dx}{dy}$.

$$\frac{dx}{dy}=\frac{x+2y^2}{y}$$

(2) **Step 2: Rewrite in Linear Form** Split the fraction on the right side:

$$\frac{dx}{dy} = \frac{x}{y} + \frac{2y^2}{y}$$

$$\frac{dx}{dy} = \frac{1}{y}x + 2y$$

Bring the x term to the left side to match the standard form $\frac{dx}{dy} + P_1 x = Q_1$:

$$\frac{dx}{dy} - \frac{1}{y}x = 2y$$

(3) **Step 3: Identify** P_1 Comparing with the standard form, the coefficient of x is:

$$P_1 = -\frac{1}{y}$$

(4) **Step 4: Calculate the Integrating Factor**

$$\text{I.F.} = e^{\int P_1 \, dy}$$

$$\text{I.F.} = e^{\int -\frac{1}{y} dy}$$

$$\text{I.F.} = e^{-\ln y}$$

Using the logarithmic property $a \ln x = \ln(x^a)$:

$$\text{I.F.} = e^{\ln(y^{-1})}$$

Using the property $e^{\ln k} = k$:

$$\text{I.F.} = y^{-1} = \frac{1}{y}$$

www.cuet.pw www.cbse.page

3. Short Tricks (Recognition)

- **Degree Check:** Look at the powers of the variables in the original equation $(x + 2y^2)dy = y\,dx$.
  - The variable y appears with power 2 (y^2).
  - The variable x appears only with power 1.

- **Rule of Thumb:** The differential equation is usually linear in the variable that has the **lower degree** (power 1). Since x is degree 1, solve for $\frac{dx}{dy}$.
- Once in the form $\frac{dx}{dy} - \frac{1}{y}x = \ldots$, the term next to x is your P. Don't forget the negative sign!

www.cuet.pw www.cbse.page

4. Final Answer

- The Integrating Factor is $\frac{1}{y}$.
- This corresponds to option **(b)**.

NCET 2023 PYQ 26

Match List I and List II for properties of definite integrals

www.cuet.pw www.cbse.page

LIST I

(A) $\int_a^b f(x)\,dx$

(B) $\int_0^a f(x)\,dx$

(C) $\int_{-a}^a f(x)\,dx$ if f is an even function

(D) $\int_0^{2a} f(x)\,dx$

www.cuet.pw www.cbse.page

LIST II

(I) $2\int_0^a f(x)\,dx$

(II) $\int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

(III) $\int_a^b f(a+b-x)\,dx$

(IV) $\int_0^a f(a-x)\,dx$

www.cuet.pw www.cbse.page

Choose the correct answer from the options given below:

1. A-III, B-IV, C-I, D-II
2. A-IV, B-I, C-II, D-III
3. A-III, B-IV, C-II, D-I
4. A-II, B-IV, C-III, D-I

Concept Builder

Correct Answer

The correct answer is: **1. A-III, B-IV, C-I, D-II**

www.cuet.pw www.cbse.page

Detailed Concepts and Step-by-Step Solution

This question requires matching standard properties of definite integrals. Let's analyze and prove each match one by one.

- **Matching (A):** $\int_a^b f(x)\,dx$
  - **This is Property P4 (often called the 'King Property'):**
    $$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$
  - **Proof:** Let $I = \int_a^b f(a+b-x)\,dx$. We use a substitution. Let $t = a+b-x$. Then, $dt = -dx$. We must also change the limits of integration:
    * When $x = a$, $t = a+b-a = b$.
    * When $x = b$, $t = a+b-b = a$.

    Substituting these into I:
    $$I = \int_b^a f(t)\,(-dt)$$
    We can use the negative sign to flip the limits of integration (from $\int_b^a$ to $\int_a^b$):
    $$I = \int_a^b f(t)\,dt$$
    By the property that the variable of integration does not matter, $\int_a^b f(t)\,dt = \int_a^b f(x)\,dx$. Thus, $\int_a^b f(a+b-x)\,dx = \int_a^b f(x)\,dx$.
  - **Conclusion:** This matches item (III) from List II.
  - **Match: A → III**

- **Matching (B):** $\int_0^a f(x)\,dx$
  - **This is Property P3 (a special case of the King Property):**
    $$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$
  - **Proof:** This is a direct application of the property (A → III) we just proved. Let $a = 0$ and $b = a$ in the property
    $$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx.$$
    $$\int_0^a f(x)\,dx = \int_0^a f(0+a-x)\,dx$$
    $$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$
  - **Conclusion:** This matches item (IV) from List II.
  - **Match: B → IV**
- **Matching (C):** $\int_{-a}^a f(x)\,dx$ **if** f **is an even function**
  - **This is Property P7 (Even/Odd Function Property):** By definition, an even function is one where $f(-x) = f(x)$.
  - **Proof:** First, we split the integral at $x = 0$:
    $$\int_{-a}^a f(x)\,dx = \int_{-a}^0 f(x)\,dx + \int_0^a f(x)\,dx$$

    Let's focus on the first part, $I_1 = \int_{-a}^0 f(x)\,dx$. We use a substitution.

    Let $t = -x$. Then, $dt = -dx$. We change the limits:
    * When $x = -a$, $t = -(-a) = a$.
    * When $x = 0$, $t = 0$.

    Substituting these into I_1:
    $$I_1 = \int_a^0 f(-t)\,(-dt) = \int_0^a f(-t)\,dt$$

    Since f is an even function, we know $f(-t) = f(t)$.
    $$I_1 = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$$

Now, substitute this back into our original split integral:

$$\int_{-a}^{a} f(x)\,dx = \left(\int_{0}^{a} f(x)\,dx\right) + \int_{0}^{a} f(x)\,dx$$

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

- **Conclusion:** This matches item (I) from List II.
- **Match: C → I**

▷ **Matching (D):** $\int_{0}^{2a} f(x)\,dx$

- **This is Property P6 (Splitting Property):**
- **Proof:** We split the integral at the midpoint, $x = a$:

$$\int_{0}^{2a} f(x)\,dx = \int_{0}^{a} f(x)\,dx + \int_{a}^{2a} f(x)\,dx$$

Let's focus on the second part, $I_2 = \int_{a}^{2a} f(x)\,dx$. We use a substitution. Let $t = 2a - x$. Then, $dt = -dx$. We change the limits:

* When $x = a$, $t = 2a - a = a$.
* When $x = 2a$, $t = 2a - 2a = 0$.

Substituting these into I_2:

$$I_2 = \int_{a}^{0} f(2a - t)\,(-dt) = \int_{0}^{a} f(2a - t)\,dt$$

Changing the variable back to x:

$$I_2 = \int_{0}^{a} f(2a - x)\,dx$$

Now, substitute this back into our original split integral:

$$\int_{0}^{2a} f(x)\,dx = \int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(2a - x)\,dx$$

- **Conclusion:** This matches item (II) from List II.
- **Match: D → II**

www.cuet.pw www.cbse.page

Final Matching Summary

- (A) $\int_a^b f(x)\,dx$ matches (III) $\int_a^b f(a+b-x)\,dx$
- (B) $\int_0^a f(x)\,dx$ matches (IV) $\int_0^a f(a-x)\,dx$
- (C) $\int_{-a}^a f(x)\,dx$ (even f) matches (I) $2\int_0^a f(x)\,dx$
- (D) $\int_0^{2a} f(x)\,dx$ matches (II) $\int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

Therefore, the correct matching is **A-III, B-IV, C-I, D-II**. This corresponds to **Option 1**.

www.cuet.pw www.cbse.page

Short Tricks and Memory Aids

- **(A → III) The King Property:** This is the most powerful property. Remember it as: the integral of $f(x)$ is the same as the integral of f(sum of limits $- x$).
- **(B → IV) The 0-to-a Property:** This is the most *common* version of the King Property. The sum of limits is $(0 + a) = a$. So $f(x)$ becomes $f(a-x)$.
- **(C → I) The Even/Odd Property:** Think graphically.
  - An **even** function (like $y = x^2$ or $y = \cos(x)$) is symmetric about the y-axis.
  - The area from $-a$ to 0 is identical to the area from 0 to a.
  - Therefore, the total area $\int_{-a}^a$ is just **two times** the area from 0 to a $\left(2\int_0^a\right)$
  - (Bonus trick: If f was an **odd** function (like $y = x^3$ or $y = \sin(x)$), the area from $-a$ to 0 would be negative and exactly cancel the positive area from 0 to a. In that case, $\int_{-a}^a f(x)\,dx = 0$.)
- **(D → II) The 0-to-2a Property:** Remember this as "splitting the integral in half". You break the journey from 0 to $2a$ into two parts: 0 to a and a to $2a$. The property shows how to transform the second part $\int_a^{2a}$ into an integral that also has limits 0 to a, which is

$$\int_0^a f\,(2a-x)\,dx.$$

NCET 2023 PYQ 27

$$\int 3^x e^x \, dx =$$

www.cuet.pw www.cbse.page

(a) $(3e)^x + c$ (b) $\dfrac{(3e)^x}{\log 3e} + c$ (c) $x(3e)^{x-1} + c$ (d) $\dfrac{(3e)^{x+1}}{x+1} + c$

Concept Builder

1. Core Concepts

- **Exponent Rule (Product of Powers):** The most important concept here is the exponent rule that allows you to combine two powers with the same exponent:

$$a^x \cdot b^x = (a \cdot b)^x$$

- **Integration of an Exponential Function:** The standard formula for integrating an exponential function a^x (where a is a constant) is:

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

(Note: $\ln a$ is the natural logarithm of a, also written as $\log_e a$).

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

(1) **Step 1: Simplify the Integrand** We are asked to integrate $\int 3^x e^x \, dx$. Using the product of powers rule, we combine 3^x and e^x:

$$3^x e^x = (3 \cdot e)^x = (3e)^x$$

So, the integral becomes:

$$\int (3e)^x \, dx$$

(2) **Step 2: Apply the Integration Formula** This integral is now in the standard form $\int a^x \, dx$, where the constant $a = 3e$. We apply the formula $\int a^x \, dx = \dfrac{a^x}{\ln a} + C$:

$$\int (3e)^x \, dx = \frac{(3e)^x}{\ln (3e)} + C$$

(3) **Step 3: Match with Options** The options use the notation $\log 3e$. In many contexts, log written without a base (especially in calculus) implies the natural logarithm (ln). Therefore, $\ln(3e)$ is the same as $\log(3e)$. Our result is:

$$\frac{(3e)^x}{\log 3e} + C$$

www.cuet.pw www.cbse.page

3. Short Tricks

- **Combine First:** The "trick" is to not overthink the problem. Seeing a product might make you think of complex methods like integration by parts, but the first step should always be algebraic simplification.
- **Power Rule vs. Exponential Rule:** The biggest trap is to misuse the power rule $\left(\int x^n dx\right)$. Remember:
  - ✓ Power Rule: Variable is in the base (x^3).
  - ✓ Exponential Rule: Variable is in the exponent (3^x).

  This problem uses the exponential rule. Options (c) and (d) incorrectly apply variations of the power rule.

www.cuet.pw www.cbse.page

4. Final Answer

- The correct integral is $\frac{(3e)^x}{\log 3e} + C$.
- This corresponds to option **(b)**.

CUET 2023 PYQ 28

$$\int \frac{x+\sqrt{x}}{2\sqrt{x}}\,dx =$$

www.cuet.pw www.cbse.page

(a) $2x\sqrt{x}+3x+C$ (b) $2\sqrt{x}+C$ (c) $x\sqrt{x}+x+C$ (d) $\frac{x\sqrt{x}}{3}+\frac{x}{2}+C$

Concept Builder

www.cuet.pw www.cbse.page

- **Simplification before Integration:** Complex algebraic fractions inside an integral are often best handled by simplifying the expression first. The strategy "Divide and Conquer" applies here: break the fraction into separate terms.

$$\frac{A+B}{C}=\frac{A}{C}+\frac{B}{C}$$

- **Laws of Exponents:** To integrate terms involving roots, convert them to fractional exponents.

$$\sqrt{x}=x^{1/2}$$

$$\frac{x}{\sqrt{x}}=x^{1-(1/2)}=x^{1/2}$$

- **Power Rule for Integration:** The standard formula for integrating powers of x is:

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C \quad (\text{where } n\neq -1)$$

- **Integration of Constants:**

$$\int k\,dx=kx+C$$

www.cuet.pw www.cbse.page

Step-by-Step Solution

- **Step 1: Simplify the Integrand.** The given integral is:

$$I = \int \frac{x + \sqrt{x}}{2\sqrt{x}} dx$$

Split the numerator over the denominator:

$$I = \int \left(\frac{x}{2\sqrt{x}} + \frac{\sqrt{x}}{2\sqrt{x}} \right) dx$$

- **Step 2: Apply Exponent Rules.** Simplify each term individually:
  - First term: $\frac{x}{2\sqrt{x}} = \frac{1}{2} \cdot \frac{x^1}{x^{1/2}} = \frac{1}{2} x^{1-1/2} = \frac{1}{2} x^{1/2}$
  - Second term: $\frac{\sqrt{x}}{2\sqrt{x}} = \frac{1}{2}$

So the integral becomes:

$$I = \int \left(\frac{1}{2} x^{1/2} + \frac{1}{2} \right) dx$$

- **Step 3: Integrate Term by Term.** Factor out the constant $\frac{1}{2}$:

$$I = \frac{1}{2} \int x^{1/2} dx + \frac{1}{2} \int 1 \, dx$$

Apply the Power Rule ($n = \frac{1}{2}$):

$$I = \frac{1}{2} \left(\frac{x^{(1/2)+1}}{\frac{1}{2}+1} \right) + \frac{1}{2}(x) + C$$

$$I = \frac{1}{2} \left(\frac{x^{3/2}}{\frac{3}{2}} \right) + \frac{x}{2} + C$$

- **Step 4: Final Simplification.**

$$I = \frac{1}{2} \cdot \frac{2}{3} x^{3/2} + \frac{x}{2} + C$$

$$I = \frac{1}{3} x^{3/2} + \frac{x}{2} + C$$

Since $x^{3/2} = x^1 \cdot x^{1/2} = x\sqrt{x}$, we can write:

$$I = \frac{x\sqrt{x}}{3} + \frac{x}{2} + C$$

- **Correct Option:** (d) $\frac{x\sqrt{x}}{3} + \frac{x}{2} + C$

www.cuet.pw www.cbse.page

Alternative Method (Substitution)

- **Substitution:** Let $\sqrt{x} = t$. Then $x = t^2$ and differentiating gives $dx = 2t\,dt$.
- **Transform Integral:** Substitute these into the original expression:

$$I = \int \frac{t^2 + t}{2t}(2t\,dt)$$

Cancel $2t$ from numerator and denominator:

$$I = \int (t^2 + t)\,dt$$

- **Integrate:**

$$I = \frac{t^3}{3} + \frac{t^2}{2} + C$$

- **Back-Substitute:** Replace $t = \sqrt{x}$:

$$I = \frac{(\sqrt{x})^3}{3} + \frac{(\sqrt{x})^2}{2} + C$$

$$I = \frac{x\sqrt{x}}{3} + \frac{x}{2} + C$$

www.cuet.pw www.cbse.page

Short Trick (Differentiation Check)

- **Concept:** Integration is the reverse of differentiation. You can differentiate the options to see which one yields the question's integrand.
- **Check Option (d):**

$$y = \frac{1}{3}x^{3/2} + \frac{1}{2}x$$

$$y' = \frac{1}{3} \cdot \frac{3}{2}x^{1/2} + \frac{1}{2}$$

$$y' = \frac{1}{2}\sqrt{x} + \frac{1}{2} = \frac{\sqrt{x} + 1}{2}$$

- **Check Integrand:**

$$\frac{x+\sqrt{x}}{2\sqrt{x}} = \frac{\sqrt{x}(\sqrt{x}+1)}{2\sqrt{x}} = \frac{\sqrt{x}+1}{2}$$

- **Conclusion:** Since the derivative matches the integrand, Option (d) is correct.

CUET 2023 PYQ 29

If $\int_0^1 \frac{a\sqrt{x}}{\left(x^{3/2}+1\right)^2}\,dx = 1$, then a is equal to

www.cuet.pw www.cbse.page

(a) 2 (b) 3 (c) 1 (d) 4

Concept Builder

www.cuet.pw www.cbse.page

Problem Statement

Given the equation:

$$\int_0^1 \frac{a\sqrt{x}}{\left(x^{3/2}+1\right)^2}dx = 1$$

We need to find the value of a.

www.cuet.pw www.cbse.page

Method 1: Substitution Method (Detailed)

This is the standard approach to solving integrals where the numerator contains the derivative (or a multiple of it) of the inner function in the denominator.

- **Identify the Substitution:** Observe that the denominator contains the term $\left(x^{3/2}+1\right)$. Let us differentiate $x^{3/2}$:

$$\frac{d}{dx}\left(x^{3/2}\right) = \frac{3}{2}x^{1/2} = \frac{3}{2}\sqrt{x}$$

  Notice that $\sqrt{x}$ is present in the numerator. This suggests the substitution $u = x^{3/2}+1$.

- **Perform the Substitution:** Let $u = x^{3/2}+1$. Then, differentiating with respect to x:

$$du = \frac{3}{2}\sqrt{x}\,dx$$

  Rearranging to isolate the term in our integral ($\sqrt{x}\,dx$):

$$\frac{2}{3}du = \sqrt{x}\,dx$$

- **Change the Limits of Integration:** Since we are changing the variable from x to u, we must update the limits.
  - Lower Limit: When $x = 0$, $u = 0^{3/2} + 1 = 1$.
  - Upper Limit: When $x = 1$, $u = 1^{3/2} + 1 = 2$.
- **Rewrite the Integral:** Substitute u, du, and the new limits into the original equation:

$$\int_0^1 \frac{a\sqrt{x}}{\left(x^{3/2}+1\right)^2} dx = a\int_1^2 \frac{1}{u^2} \cdot \left(\frac{2}{3} du\right)$$

$$= \frac{2a}{3}\int_1^2 u^{-2}\, du$$

- **Solve the Integral:** Using the power rule $\int u^n du = \frac{u^{n+1}}{n+1}$:

$$\frac{2a}{3}\left[\frac{u^{-1}}{-1}\right]_1^2 = \frac{2a}{3}\left[-\frac{1}{u}\right]_1^2$$

- **Apply Limits:**

$$= -\frac{2a}{3}\left(\frac{1}{2} - \frac{1}{1}\right)$$

$$= -\frac{2a}{3}\left(\frac{1}{2} - 1\right)$$

$$= -\frac{2a}{3}\left(-\frac{1}{2}\right)$$

$$= \frac{a}{3}$$

www.cuet.pw www.cbse.page

Method 2: Short Trick (Inspection)

For competitive exams, you can recognize the form $\int \frac{f'(x)}{[f(x)]^n} dx$.

- **Concept:**

$$\int \frac{f'(x)}{\left(f(x)\right)^2} dx = -\frac{1}{f(x)}$$

- **Application:** Here, let $f(x) = x^{3/2} + 1$. Then $f'(x) = \frac{3}{2}\sqrt{x}$. The integral has $a\sqrt{x}$ in the numerator. We need $\frac{3}{2}\sqrt{x}$ to match $f'(x)$.

Rewrite the numerator:

$$a\sqrt{x} = a \cdot \frac{2}{3} \cdot \left(\frac{3}{2}\sqrt{x}\right)$$

So the integral becomes:

$$\frac{2a}{3}\int_0^1 \frac{\frac{3}{2}\sqrt{x}}{\left(x^{3/2}+1\right)^2}dx$$

Directly apply the result $[-1/f(x)]$:

$$= \frac{2a}{3}\left[-\frac{1}{x^{3/2}+1}\right]_0^1$$

Evaluate mentally:

$$\frac{2a}{3}\left(-\frac{1}{2}-(-1)\right) = \frac{2a}{3}\cdot\frac{1}{2} = \frac{a}{3}$$

www.cuet.pw www.cbse.page

Final Calculation

We have calculated the value of the integral to be $\frac{a}{3}$. Given that the integral equals 1:

$$\frac{a}{3} = 1$$
$$a = 3$$

www.cuet.pw www.cbse.page

The correct option is: (b) 3

Practice Question 1 :

If $\int_0^1 \frac{a\sqrt{x}}{\left(x^{1/2}+1\right)^2}dx = 1$, then a is equal to

www.cuet.pw www.cbse.page

(a) 2 (b) 3 (c) 1 (d) 4

Concept Builder

Let the given integral be I:

$$I = \int_0^1 \frac{a\sqrt{x}}{\left(x^{1/2}+1\right)^2} dx$$

www.cuet.pw www.cbse.page

Step 1: Substitution

Let us use the substitution $u = \sqrt{x}$.

- Squaring both sides, $x = u^2$.
- Differentiating with respect to u, we get $dx = 2u\,du$.

Now, we change the limits of integration:

- Lower limit: When $x = 0$, $u = \sqrt{0} = 0$.
- Upper limit: When $x = 1$, $u = \sqrt{1} = 1$.

www.cuet.pw www.cbse.page

Step 2: Transforming the Integral

Substituting the values of x, dx, and the limits into the integral:

$$I = \int_0^1 \frac{a \cdot u}{(u+1)^2} \cdot (2u)\, du$$

$$I = 2a \int_0^1 \frac{u^2}{(u+1)^2} du$$

www.cuet.pw www.cbse.page

Step 3: Algebraic Manipulation

To integrate $\frac{u^2}{(u+1)^2}$, we rearrange the numerator to make it divisible by the denominator:

$$\frac{u^2}{(u+1)^2} = \frac{(u+1-1)^2}{(u+1)^2}$$

$$= \frac{(u+1)^2 - 2(u+1) + 1}{(u+1)^2}$$

$$= 1 - \frac{2}{u+1} + \frac{1}{(u+1)^2}$$

www.cuet.pw www.cbse.page

Step 4: Integration

Now, substituting the simplified expression back into the integral:

$$I = 2a \int_0^1 \left(1 - \frac{2}{u+1} + (u+1)^{-2}\right) du$$

Integrating term by term with respect to u:

$$I = 2a \left[u - 2\ln|u+1| + \frac{(u+1)^{-1}}{-1}\right]_0^1$$

$$I = 2a \left[u - 2\ln(u+1) - \frac{1}{u+1}\right]_0^1$$

www.cuet.pw www.cbse.page

Step 5: Applying Limits

Evaluate the expression at the upper and lower limits:

- At upper limit ($u = 1$):

$$\left(1 - 2\ln(1+1) - \frac{1}{1+1}\right) = \left(1 - 2\ln(2) - \frac{1}{2}\right) = \frac{1}{2} - 2\ln(2)$$

- At lower limit ($u = 0$):

$$\left(0 - 2\ln(0+1) - \frac{1}{0+1}\right) = (0 - 0 - 1) = -1$$

Calculate the final value by subtracting the lower limit value from the upper limit value:

$$I = 2a \left[\left(\frac{1}{2} - 2\ln(2)\right) - (-1)\right]$$

$$I = 2a \left(\frac{1}{2} + 1 - 2\ln(2)\right)$$

$$I = 2a \left(\frac{3}{2} - 2\ln(2)\right)$$

$$I = 3a - 4a\ln(2)$$

www.cuet.pw www.cbse.page

Final Answer

$$\int_0^1 \frac{a\sqrt{x}}{\left(x^{1/2}+1\right)^2} dx = a(3 - 4\ln(2))$$

www.cuet.pw www.cbse.page

Determination of the Constant a

From the previous evaluation, the value of the definite integral is:

$$I = \int_0^1 \frac{a\sqrt{x}}{\left(x^{1/2}+1\right)^2} dx = a\,(3 - 4\ln(2))$$

www.cuet.pw www.cbse.page

As per question given: $\int_0^1 \frac{a\sqrt{x}}{\left(x^{1/2}+1\right)^2}\, dx = 1$

Substituting our result:

$$a\,(3 - 4\ln(2)) = 1$$

Solving for a:

$$a = \frac{1}{3 - 4\ln(2)}$$

www.cuet.pw www.cbse.page

Numerical Approximation

We can calculate the approximate numerical coefficient using $\ln(2) \approx 0.693147$:

- $4\ln(2) \approx 2.772588$
- Denominator: $3 - 2.772588 = 0.227412$

If the integral equals 1, then:

$$a \approx \frac{1}{0.227412} \approx 4.3973$$

www.cuet.pw www.cbse.page

www.cuet.pw www.cbse.page

Exam Insight & Possible Typing Correction

The answer $a = \frac{1}{3 - 4\ln(2)}$ is mathematically correct for the question **exactly as written**. However, in competitive exams, answers are usually integers or simple fractions.

www.cuet.pw www.cbse.page

Points to remember

- **Spot the Derivative:** Always check if the derivative of a messy term is present elsewhere in the integrand. If you see $\sqrt{x}$ inside a bracket and $\frac{1}{\sqrt{x}}$ outside, use substitution immediately.

- **Degree Match Trick:** If numerator degree = denominator degree (after sub), always perform division or add/subtract constants to split the fraction.
- **Limit Check:** Don't forget to update limits. A common mistake is solving the indefinite integral and using original x limits on the t variable.

Practice Question 2 :

If $\int_0^1 \frac{a\sqrt{x}dx}{\left(\frac{1}{3}x^{3/2}+1\right)^2} = 1$, then a is equal to

www.cuet.pw www.cbse.page

(a) 2 (b) 3 (c) 1 (d) 4

Concept Builder

www.cuet.pw www.cbse.page

- **Step 1: Set up the Substitution.** Let the term inside the denominator's power be t.

$$t = \frac{1}{3}x^{3/2}+1$$

Differentiate with respect to x:

$$\frac{dt}{dx} = \frac{1}{3}\cdot\frac{3}{2}x^{((3/2)-1)}+0$$

$$\frac{dt}{dx} = \frac{1}{2}x^{1/2} = \frac{1}{2}\sqrt{x}$$

Rearrange to isolate the numerator term $\sqrt{x}dx$:

$$2dt = \sqrt{x}dx$$

- **Step 2: Change the Limits of Integration.**
  - **Lower Limit** ($x = 0$)**:**

$$t = \frac{1}{3}(0)^{\frac{3}{2}}+1 = 1$$

- **Upper Limit ($x = 1$):**

$$t = \frac{1}{3}(1)^{3/2} + 1 = \frac{1}{3} + 1 = \frac{4}{3}$$

- **Step 3: Transform the Integral.** Substitute $\sqrt{x}dx = 2dt$ and the new limits into the given equation:

$$\int_1^{4/3} \frac{a(2dt)}{t^2} = 1$$

Factor out the constants:

$$2a\int_1^{4/3} t^{-2}dt = 1$$

- **Step 4: Integrate and Solve.** Using the standard integral $\int t^{-2}dt = -\frac{1}{t}$:

$$2a\left[-\frac{1}{t}\right]_1^{4/3} = 1$$

Apply the limits (Upper - Lower):

$$2a\left[\left(-\frac{1}{4/3}\right) - \left(-\frac{1}{1}\right)\right] = 1$$

Simplify the fractions:

$$2a\left[-\frac{3}{4} + 1\right] = 1$$

$$2a\left[\frac{1}{4}\right] = 1$$

$$\frac{a}{2} = 1$$

$$a = 2$$

- **Correct Option:** (a) 2

www.cuet.pw www.cbse.page

Short Trick (Inspection Method)

- If you recognize the form $\int \frac{k \cdot f'(x)}{[f(x)]^2}dx$, the result is simply $-\frac{k}{f(x)}$.

- **Application:**

1. Note that derivative of $\left(\frac{1}{3}x^{3/2}+1\right)$ is $\frac{1}{2}\sqrt{x}$.

2. The numerator is $a\sqrt{x}$, which is $2a\left(\frac{1}{2}\sqrt{x}\right)$. Here, our constant multiplier $k=2a$.

3. Directly write the integrated result: $\left[-\frac{2a}{\frac{1}{3}x^{3/2}+1}\right]_0^1 = 1.$

4. Plug in limits:

$$-2a\left(\frac{1}{4/3}-\frac{1}{1}\right)=1$$

$$-2a\left(\frac{3}{4}-1\right)=1 \implies -2a\left(-\frac{1}{4}\right)=1 \implies \frac{a}{2}=1 \implies a=2$$

CUET 2023 PYQ 30

Integral $\int e^x\left(1+x^2\right)dx =$

www.cuet.pw www.cbse.page

(1) $e^x\left(\frac{x^3}{3}+x\right)+c$

(2) $\left(x^2+2x\right)e^x+c$

(3) $\left(x^2-2x+3\right)e^x+c$

(4) $2xe^x+c$

(c is a constant)

Concept Builder

www.cuet.pw www.cbse.page

Key Concepts and Formulae

- **Integration by Parts (IBP):** This is the product rule for integration. If u and v are functions of x:

$$\int u\,v\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$$

  Or simply: $\int u\,dv = uv - \int v\,du$.

- **ILATE Rule:** To choose the first function (u), we follow the priority order: **I** - Inverse Trigonometric, **L** - Logarithmic, **A** - Algebraic, **T** - Trigonometric, **E** - Exponential. Here, $1+x^2$ is Algebraic and e^x is Exponential. So, $u = 1+x^2$.

- **Special Integral Form:** A very important property for integrals involving e^x:

$$\int e^x\left[f(x)+f'(x)\right]dx = e^x f(x)+c$$

www.cuet.pw www.cbse.page

Step-by-Step Solution (Method 1: Integration by Parts)

- **Step 1: Assign u and dv** Let $I = \int (1+x^2)e^x\,dx$. Using ILATE, take $u = (1+x^2)$ and $dv = e^x dx$. Then, differentiate u: $du = 2x\,dx$. And integrate dv: $v = \int e^x dx = e^x$.
- **Step 2: Apply the formula**

$$I = (1+x^2)e^x - \int e^x(2x)\,dx$$

$$I = (1+x^2)e^x - 2\int xe^x\,dx$$

- **Step 3: Apply Integration by Parts again** For the remaining integral $\int xe^x dx$: Take $u_2 = x$ and $dv_2 = e^x dx$. Then $du_2 = dx$ and $v_2 = e^x$.

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x$$

- **Step 4: Substitute back into the main equation** Substitute the result from Step 3 into the equation from Step 2:

$$I = (1+x^2)e^x - 2\left[xe^x - e^x\right] + c$$

- **Step 5: Simplify**

$$I = (1+x^2)e^x - 2xe^x + 2e^x + c$$

Factor out e^x:

$$I = e^x\left(1+x^2-2x+2\right) + c$$

$$I = e^x\left(x^2-2x+3\right) + c$$

- **Conclusion** This matches **Option 3.**

www.cuet.pw www.cbse.page

Method 2: Reverse Engineering (Checking Derivatives)

- Since integration is the reverse of differentiation, the derivative of the correct option must equal the integrand $e^x(1+x^2)$.
- **Check Option 3:** $y = e^x(x^2-2x+3)$ Apply the Product Rule $(uv' + u'v)$:

$$\frac{dy}{dx} = e^x\frac{d}{dx}(x^2-2x+3) + (x^2-2x+3)\frac{d}{dx}(e^x)$$

$$= e^x(2x-2) + (x^2-2x+3)e^x$$

Take e^x common:

$$= e^x(2x-2+x^2-2x+3)$$

$$= e^x(x^2+1)$$

- This matches the question perfectly. Thus, Option 3 is correct.

www.cuet.pw www.cbse.page

Short Trick (Mental Inspection)

- **Pattern Recognition:** Look for the form $e^x[f(x)+f'(x)]$. The integrand is $e^x[x^2+1]$. We need to split (x^2+1) into a function and its derivative.
- Since we have an x^2, $f(x)$ must be quadratic. Let's guess $f(x)=x^2-2x+3$. Then $f'(x)=2x-2$. Add them up:

$$(x^2-2x+3)+(2x-2)=x^2+1$$

It fits exactly!

- Therefore, the answer is immediately $e^x f(x)=e^x(x^2-2x+3)$.

CUET 2023 PYQ 31

If $I = \int_0^{\pi} x \sin x \, dx$, then

www.cuet.pw www.cbse.page

(1) $I = -\pi$ (2) $I = 2\pi$ (3) $I = 2$ (4) $I = \pi$

Concept Builder

www.cuet.pw www.cbse.page

Key Concepts and Formulae

- **Integration by Parts (Definite Integral):** When the integrand is a product of two different types of functions (e.g., algebraic and trigonometric), we use the product rule for integration:

$$\int_a^b u(x)\,v'(x)\,dx = [u(x)\,v(x)]_a^b - \int_a^b v(x)\,u'(x)\,dx$$

  Where u is the first function and v' is the second function.

- **ILATE Rule:** To decide which function is u, we follow the priority order: **I** - Inverse Trig, **L** - Logarithmic, **A** - Algebraic, **T** - Trigonometric, **E** - Exponential. Here, x is **A**lgebraic and $\sin x$ is **T**rigonometric. Since A comes before T, we choose $u = x$.

- **Property of Definite Integrals (King's Rule):**

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

  This property is extremely powerful for simplifying definite integrals involving limits like 0 to π.

www.cuet.pw www.cbse.page

Step-by-Step Solution (Method 1: Integration by Parts)

- **Step 1: Assign u and dv** Let $I = \int_0^{\pi} x \sin x \, dx$. Using ILATE:

$$u = x \implies du = dx$$

$$dv = \sin x\,dx \implies v = \int \sin x\,dx = -\cos x$$

- **Step 2: Apply the Integration by Parts Formula**

$$I = [x(-\cos x)]_0^{\pi} - \int_0^{\pi} (-\cos x)\, dx$$

$$I = [-x\cos x]_0^{\pi} + \int_0^{\pi} \cos x\, dx$$

- **Step 3: Evaluate the First Term limits**

$$[-x\cos x]_0^{\pi} = (-\pi\cos\pi) - (-0\cos 0)$$

Since $\cos\pi = -1$:

$$= (-\pi)(-1) - 0 = \pi$$

- **Step 4: Evaluate the Remaining Integral**

$$\int_0^{\pi} \cos x\, dx = [\sin x]_0^{\pi}$$

$$= \sin\pi - \sin 0$$

$$= 0 - 0 = 0$$

- **Step 5: Combine Results**

$$I = \pi + 0 = \pi$$

- **Correct option :** The value of the integral is π **(Option 4)**.

www.cuet.pw www.cbse.page

Alternative Method (Method 2: King's Property)

- **Step 1: Apply the Property** Let $I = \int_0^{\pi} x\sin x\, dx$ ... **(i)** Using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:

$$I = \int_0^{\pi} (\pi - x)\sin(\pi - x)\, dx$$

Since $\sin(\pi - x) = \sin x$:

$$I = \int_0^{\pi} (\pi - x)\sin x\, dx$$

$$I = \int_0^{\pi} \pi\sin x\, dx - \int_0^{\pi} x\sin x\, dx$$

- **Step 2: Substitute I back** Notice that the second term is the original integral I:

$$I = \pi \int_0^{\pi} \sin x \, dx - I$$

- **Step 3: Solve for I** Move I to the left side:

$$2I = \pi \int_0^{\pi} \sin x \, dx$$

$$2I = \pi \left[-\cos x \right]_0^{\pi}$$

$$2I = -\pi [\cos \pi - \cos 0]$$

$$2I = -\pi [-1 - 1] = -\pi [-2] = 2\pi$$

$$I = \pi$$

- This confirms the previous result and is often faster as it avoids integration by parts directly on the product.

www.cuet.pw www.cbse.page

Short Trick (Tabular Method / DI Method)

- For integrals of the form $\int x \cdot (\text{trig}) \, dx$, simply differentiate the algebraic part to zero and integrate the trig part, multiplying diagonals with alternating signs $(+, -)$.

Sign	**Diff** (x)	**Int** $(\sin x)$
$+$	x	$\sin x$
$-$	1	$-\cos x$
$+$	0	$-\sin x$

- **Calculation:**

$$= (+)(x)(-\cos x) - (1)(-\sin x)$$

$$= -x \cos x + \sin x$$

Apply limits 0 to π:

$$= [-\pi(-1) + 0] - [0 + 0] = \pi$$

CUET 2023 PYQ 32

If $\int_0^1 \left(3x^2+2x+5k\right)dx = 0$, then the value of k is :

www.cuet.pw www.cbse.page

(1) $\frac{3}{5}$ (2) $-\frac{2}{5}$ (3) $\frac{2}{5}$ (4) $\frac{1}{5}$

Concept Builder

www.cuet.pw www.cbse.page

Detailed Explanation and Solution

www.cuet.pw www.cbse.page

Key Concepts Required

- **Power Rule of Integration:** The integral of a power of x is given by:

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad (\text{where } n \neq -1)$$

Constant terms integrate as: $\int k\,dx = kx$.

- **Fundamental Theorem of Calculus (Definite Integrals):** If $F(x)$ is the anti-derivative of $f(x)$, then:

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

www.cuet.pw www.cbse.page

Step-by-Step Solution

- **Step 1: Perform the Integration** We need to find the anti-derivative of the function $f(x) = 3x^2+2x+5k$. Integrating term by term with respect to x:

$$\int \left(3x^2+2x+5k\right)dx = 3\left(\frac{x^3}{3}\right)+2\left(\frac{x^2}{2}\right)+5k\,(x)$$

Simplify the coefficients:

$$= x^3 + x^2 + 5kx$$

- **Step 2: Apply the Limits** Now, apply the lower limit (0) and the upper limit (1) to the result:

$$\left[x^3 + x^2 + 5kx\right]_0^1$$

Substitute $x = 1$ (Upper Limit):

$$(1)^3 + (1)^2 + 5k(1) = 1 + 1 + 5k = 2 + 5k$$

Substitute $x = 0$ (Lower Limit):

$$(0)^3 + (0)^2 + 5k(0) = 0$$

So, the value of the definite integral is:

$$(2 + 5k) - 0 = 2 + 5k$$

- **Step 3: Solve for k** The problem states that the integral equals 0.

$$2 + 5k = 0$$

Subtract 2 from both sides:

$$5k = -2$$

Divide by 5:

$$k = -\frac{2}{5}$$

www.cuet.pw www.cbse.page

Alternative Method (Short Trick for Polynomials on [0, 1])

- **Concept:** When integrating a polynomial term ax^n from 0 to 1, the result is simply the coefficient divided by the new power:
$\int_0^1 ax^n\,dx = \frac{a}{n+1}$.

- **Application:** Apply this directly to each term in $\int_0^1 \left(3x^2 + 2x + 5k\right) dx$:

(a) Term $3x^2$: Becomes $\frac{3}{2+1} = \frac{3}{3} = 1$

(b) Term $2x^1$: Becomes $\frac{2}{1+1} = \frac{2}{2} = 1$

(c) Term $5k$ (constant is x^0): Becomes $\frac{5k}{0+1} = 5k$

- **Quick Calculation:** Sum the results and set to zero:

$$1+1+5k=0$$

$$2+5k=0 \implies k=-\frac{2}{5}$$

www.cuet.pw www.cbse.page

Final Answer

The correct option is **2.** $\left(-\frac{2}{5}\right)$

CUET 2023 PYQ 33

Match List I with List II

www.cuet.pw www.cbse.page

List I

A. $\int \sin^2(2x+1)\,dx$

B. $\int \cos^2(2x+1)\,dx$

C. $\int \sin^3(2x+1)\,dx$

D. $\int \cos^3(2x+1)\,dx$

www.cuet.pw www.cbse.page

List II

I. $\frac{x}{2}+\frac{\sin(2x+1)\cos(2x+1)}{4}+C$

II. $\frac{\cos 3(2x+1)}{24}-\frac{3\cos(2x+1)}{8}+C$

III. $\frac{x}{2}-\frac{\sin(2x+1)\cos(2x+1)}{4}+C$

IV. $\frac{\sin 3(2x+1)}{24}+\frac{3\sin(2x+1)}{8}+C$

www.cuet.pw www.cbse.page

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II

(2) A-III, B-I, C-II, D-IV

(3) A-I, B-III, C-II, D-IV

(4) A-III, B-I, C-IV, D-II

Concept Builder

www.cuet.pw www.cbse.page

Key Concepts and Formulae

- **Integration of Trigonometric Powers:** To integrate powers of sin and cos like $\sin^n x$ or $\cos^n x$, we use trigonometric identities to reduce the power to linear terms.
- **Power Reduction Formulas (Square):**

  (i) $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

  (ii) $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$
- **Triple Angle Formulas (Cube):**

  (i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta \implies \sin^3\theta = \dfrac{3\sin\theta - \sin 3\theta}{4}$

  (ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta \implies \cos^3\theta = \dfrac{\cos 3\theta + 3\cos\theta}{4}$
- **Linear Substitution:** When integrating functions of $(ax+b)$, the coefficient of x appears in the denominator.

$$\int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{a} + C$$

www.cuet.pw www.cbse.page

Step-by-Step Solution

- **Analyze A:** $\int \sin^2(2x+1)\,dx$ Using $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$:

$$I_A = \int \frac{1-\cos 2(2x+1)}{2}dx = \frac{1}{2}\int [1-\cos(4x+2)]\,dx$$

$$I_A = \frac{1}{2}\left[x - \frac{\sin(4x+2)}{4}\right] = \frac{x}{2} - \frac{\sin 2(2x+1)}{8}$$

Using $\sin 2A = 2\sin A\cos A$:

$$I_A = \frac{x}{2} - \frac{2\sin(2x+1)\cos(2x+1)}{8} = \frac{x}{2} - \frac{\sin(2x+1)\cos(2x+1)}{4} + C$$

This matches **List II (III)**. So, **A - III**.

- **Analyze B:** $\int \cos^2(2x+1)\,dx$ Using $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$:

$$I_B = \int \frac{1+\cos(4x+2)}{2}dx$$

$$I_B = \frac{1}{2}\left[x + \frac{\sin(4x+2)}{4}\right] = \frac{x}{2} + \frac{\sin(2x+1)\cos(2x+1)}{4} + C$$

This matches **List II (I)**. So, **B - I**.

- **Analyze C:** $\int \sin^3(2x+1)\,dx$ Using $\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$:

$$I_C = \int \frac{3\sin(2x+1) - \sin 3(2x+1)}{4} dx$$

Integrating term by term (remembering to divide by coefficient 2 and 6 respectively):

$$I_C = \frac{1}{4}\left[3\left(\frac{-\cos(2x+1)}{2}\right) - \left(\frac{-\cos 3(2x+1)}{2\cdot 3}\right)\right]$$

$$I_C = -\frac{3\cos(2x+1)}{8} + \frac{\cos 3(2x+1)}{24} + C$$

Rearranging terms:

$$I_C = \frac{\cos 3(2x+1)}{24} - \frac{3\cos(2x+1)}{8} + C$$

This matches **List II (II)**. So, **C - II**.

- **Analyze D:** $\int \cos^3(2x+1)\,dx$ Using $\cos^3\theta = \frac{3\cos\theta + \cos 3\theta}{4}$:

$$I_D = \int \frac{3\cos(2x+1) + \cos 3(2x+1)}{4} dx$$

$$I_D = \frac{1}{4}\left[3\left(\frac{\sin(2x+1)}{2}\right) + \left(\frac{\sin 3(2x+1)}{6}\right)\right]$$

$$I_D = \frac{3\sin(2x+1)}{8} + \frac{\sin 3(2x+1)}{24} + C$$

This matches **List II (IV)**. So, **D - IV**.

- **Conclusion:** Combining results: A-III, B-I, C-II, D-IV. This corresponds to **Option 2**.

www.cuet.pw **www.cbse.page**

Short Tricks & Logic

- **Sign Logic (Square Powers):**
  - **(i)** $\int \sin^2 x$ always results in a **negative** sine term $(\frac{x}{2} - \ldots)$.

(ii) $\int \cos^2 x$ always results in a **positive** sine term $(\frac{x}{2} + \ldots)$.

(iii) Just by looking at signs in List II, you can match A $\rightarrow$ III (negative) and B $\rightarrow$ I (positive).

- **Function Logic (Cube Powers):**

(i) Integration flips the function: $\int \sin \rightarrow -\cos$ and $\int \cos \rightarrow \sin$.

(ii) So, $\int \sin^3 x$ must result in cosine terms (List II - II has cosines).

(iii) $\int \cos^3 x$ must result in sine terms (List II - IV has sines).

- **Result:** Using this logic, you can solve the match without writing a single line of integration!

- A ($\sin^2$) $\rightarrow$ Negative sign $\rightarrow$ III
- B ($\cos^2$) $\rightarrow$ Positive sign $\rightarrow$ I
- C ($\sin^3$) $\rightarrow$ Cosines $\rightarrow$ II
- D ($\cos^3$) $\rightarrow$ Sines $\rightarrow$ IV

www.cuet.pw www.cbse.page

Teaching Module :

www.cuet.pw www.cbse.page

Key Concepts and Formulas Required

- **Integration by Substitution:** When the argument of a trigonometric function is linear, like $(ax + b)$, we treat it as a variable θ.

$$\text{If } \theta = ax + b, \text{ then } dx = \frac{d\theta}{a}$$

In this problem, $\theta = 2x + 1$, so $dx = \frac{d\theta}{2}$.

- **Power Reduction Formulas (Squares):** To integrate $\sin^2\theta$ or $\cos^2\theta$, we convert them into linear cosine terms of double angles.

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

- **Triple Angle Formulas (Cubes):** To integrate $\sin^3\theta$ or $\cos^3\theta$, we use the triple angle identities.

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta \implies \sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta \implies \cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4}$$

- **Double Angle Identity (Simplification):**

$$\sin 2\theta = 2\sin\theta\cos\theta$$

www.cuet.pw www.cbse.page

Step-by-Step Solution

- **Solving A:** $\int \sin^2(2x+1)\,dx$ Let $\theta = 2x+1 \implies dx = \frac{d\theta}{2}$.

$$I_A = \int \sin^2\theta \cdot \frac{d\theta}{2} = \frac{1}{2}\int \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{4}\left[\theta - \frac{\sin 2\theta}{2}\right] = \frac{\theta}{4} - \frac{\sin 2\theta}{8}$$

Substitute $\theta = 2x+1$ back:

$$= \frac{2x+1}{4} - \frac{2\sin(2x+1)\cos(2x+1)}{8}$$

(Note: $\sin 2\theta = 2\sin\theta\cos\theta$)

$$= \frac{x}{2} + \frac{1}{4} - \frac{\sin(2x+1)\cos(2x+1)}{4}$$

Ignoring the constant $\frac{1}{4}$ (as it merges with C), this matches **List II (III)**.

Result: A → III

- **Solving B:** $\int \cos^2(2x+1)\,dx$ Using the same substitution $\theta = 2x+1$:

$$I_B = \frac{1}{2}\int \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{4}\left[\theta + \frac{\sin 2\theta}{2}\right] = \frac{x}{2} + \frac{\sin(2x+1)\cos(2x+1)}{4} + C$$

This matches **List II (I)**. **Result: B → I**

- **Solving C:** $\int \sin^3(2x+1)\,dx$ Using the identity $\sin^3\theta = \dfrac{3\sin\theta - \sin 3\theta}{4}$ and $\theta = 2x+1$:

$$I_C = \frac{1}{2}\int \left(\frac{3\sin\theta - \sin 3\theta}{4}\right) d\theta$$

$$= \frac{1}{8}\left[3(-\cos\theta) - \left(-\frac{\cos 3\theta}{3}\right)\right]$$

$$= \frac{1}{8}\left[-3\cos(2x+1) + \frac{\cos 3(2x+1)}{3}\right]$$

$$= \frac{\cos 3(2x+1)}{24} - \frac{3\cos(2x+1)}{8} + C$$

  This matches **List II (II)**. **Result: C → II**

- **Solving D:** $\int \cos^3(2x+1)\,dx$ Using the identity $\cos^3\theta = \dfrac{\cos 3\theta + 3\cos\theta}{4}$ and $\theta = 2x+1$:

$$I_D = \frac{1}{2}\int \left(\frac{\cos 3\theta + 3\cos\theta}{4}\right) d\theta$$

$$= \frac{1}{8}\left[\frac{\sin 3\theta}{3} + 3\sin\theta\right]$$

$$= \frac{\sin 3(2x+1)}{24} + \frac{3\sin(2x+1)}{8} + C$$

  This matches **List II (IV)**. **Result: D → IV**

- **Conclusion** The correct matching is: A → III B → I C → II D → IV

www.cuet.pw www.cbse.page

Alternative Method (Reverse Engineering / Differentiation)

- Instead of integrating, you can differentiate the expressions in List II to see if you get the integrand in List I.

- **Example for B:** Take Expression I: $y = \dfrac{x}{2} + \dfrac{\sin(4x+2)}{8}$ (using double angle).

$$\frac{dy}{dx} = \frac{1}{2} + \frac{1}{8}\cos(4x+2)\cdot 4 = \frac{1}{2} + \frac{1}{2}\cos(2(2x+1))$$

$$= \frac{1+\cos 2\theta}{2} = \cos^2\theta = \cos^2(2x+1)$$

  This confirms B matches I immediately without integration formulas. This method is often faster if you are good at derivatives.

www.cuet.pw www.cbse.page

Final Answer

The correct option is **2.** (A-III, B-I, C-II, D-IV)

CUET 2023 PYQ 34

The value of $\int (5x-2)^3\,dx$ is:

www.cuet.pw www.cbse.page

(1) $3(5x-2)+C$; C is constant of integration.

(2) $15(5x-2)^2+C$; C is constant of integration.

(3) $(5x-2)^4+C$; C is constant of integration.

(4) $\frac{(5x-2)^4}{20}+C$; C is constant of integration.

Concept Builder

www.cuet.pw www.cbse.page

Key Concepts and Formulas

To solve this indefinite integral, we need the standard power rule for integration and the method of substitution for linear composite functions.

- **Power Rule of Integration:** For any real number $n \neq -1$:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C$$

- **Integration by Substitution (Linear Composition):** If we have a function of the form $(ax+b)^n$, we can substitute $u = ax+b$. A direct generalization of the power rule for linear arguments is:

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}+C$$

Notice the division by 'a', which is the coefficient of x. This effectively reverses the Chain Rule from differentiation.

www.cuet.pw www.cbse.page

Step-by-Step Solution

- **Method 1: Substitution Method (Formal Approach)** Let the integral be I.

$$I = \int (5x-2)^3 \, dx$$

Step 1: Choose a substitution variable Let $u = 5x - 2$.

Step 2: Differentiate to find dx Differentiating with respect to x:

$$\frac{du}{dx} = 5$$

$$du = 5\,dx \implies dx = \frac{du}{5}$$

Step 3: Substitute into the integral Replace $(5x-2)$ with u and dx with $\frac{du}{5}$:

$$I = \int u^3 \cdot \frac{du}{5}$$

$$I = \frac{1}{5}\int u^3 \, du$$

Step 4: Integrate using Power Rule Using $\int u^n du = \frac{u^{n+1}}{n+1}$:

$$I = \frac{1}{5}\left[\frac{u^{3+1}}{3+1}\right] + C$$

$$I = \frac{1}{5}\left[\frac{u^4}{4}\right] + C$$

$$I = \frac{u^4}{20} + C$$

Step 5: Back-substitute Replace u with $(5x-2)$:

$$I = \frac{(5x-2)^4}{20} + C$$

www.cuet.pw www.cbse.page

Final Answer : The value of the integral is $\frac{(5x-2)^4}{20} + C$. The correct option is **(4)**.

www.cuet.pw www.cbse.page

Short Trick / Direct Inspection

- **The "Divide by Coefficient" Rule:** When integrating a linear function $(ax+b)$, treat the entire bracket as a single variable X, integrate normally, and then **divide the result by the coefficient of** x.

$$\text{Integral} = \frac{\text{Integrate outer power}}{\text{Coefficient of } x}$$

Application:

1. Outer integration of $(\ldots)^3$ becomes $\frac{(\ldots)^4}{4}$.

2. Coefficient of x is 5.

3. Combine:

$$\frac{1}{5} \times \frac{(5x-2)^4}{4} = \frac{(5x-2)^4}{20}$$

This allows you to solve the question mentally in seconds.

CUET 2023 PYQ 35

The value of $\int_{-1}^{1} x^2[x]\,dx$ is:

www.cuet.pw www.cbse.page

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $-\frac{1}{3}$

Concept Builder

www.cuet.pw www.cbse.page

Step-by-Step Detailed Solution

- **Understanding the Function involved**
  - The integrand contains $[x]$, which represents the **Greatest Integer Function** (GIF).
  - The definition of $[x]$ is the greatest integer less than or equal to x.
  - We need to analyze the value of $[x]$ within the limits of integration, i.e., from $x = -1$ to $x = 1$.
  - The function $[x]$ changes its value at integer points. In the interval $[-1, 1]$, the integer point is 0.

$$[x] = \begin{cases} -1 & \text{for } -1 \le x < 0 \\ 0 & \text{for } 0 \le x < 1 \end{cases}$$

  - Note: At $x = 1$, $[x] = 1$, but since it is a single point (measure zero), it does not affect the value of the definite integral.
- **Splitting the Definite Integral**
  - By the property of definite integrals:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

  - We split the integral at $x = 0$:

$$I = \int_{-1}^{1} x^2[x]\,dx = \int_{-1}^{0} x^2[x]\,dx + \int_{0}^{1} x^2[x]\,dx$$

- **Evaluating the First Part (Interval $[-1, 0]$)**
  - In the range $-1 \leq x < 0$, we have $[x] = -1$.
  - Substitute this value into the integral:

$$I_1 = \int_{-1}^{0} x^2(-1)\,dx = -\int_{-1}^{0} x^2\,dx$$

  - Integrate using power rule $\int x^n\,dx = \frac{x^{n+1}}{n+1}$:

$$I_1 = -\left[\frac{x^3}{3}\right]_{-1}^{0}$$

  - Apply limits (Upper Limit - Lower Limit):

$$I_1 = -\left(\frac{0^3}{3} - \frac{(-1)^3}{3}\right)$$

$$I_1 = -\left(0 - \frac{-1}{3}\right) = -\left(\frac{1}{3}\right) = -\frac{1}{3}$$

- **Evaluating the Second Part (Interval $[0, 1]$)**
  - In the range $0 \leq x < 1$, we have $[x] = 0$.
  - Substitute this value into the integral:

$$I_2 = \int_{0}^{1} x^2(0)\,dx = \int_{0}^{1} 0\,dx$$

  - The integral of zero is zero:

$$I_2 = 0$$

- **Calculating the Final Result**
  - Sum the two parts:

$$I = I_1 + I_2$$

$$I = -\frac{1}{3} + 0$$

$$I = -\frac{1}{3}$$

www.cuet.pw www.cbse.page

Key Concept & Graphical Insight

- **Visualizing the Area:**

- The integral $\int_{-1}^{1} x^2[x]\,dx$ represents the signed area between the curve $y = x^2[x]$ and the x-axis.
- From $x = 0$ to 1, the curve is $y = x^2(0) = 0$ (flat on the axis), so Area is 0.
- From $x = -1$ to 0, the curve is $y = -x^2$ (an inverted parabola). This area lies below the x-axis, so the integral value must be negative.
- This logic immediately eliminates options (a), (b), and (c) as they are positive. The only negative option is (d).

www.cuet.pw www.cbse.page

Final Conclusion

- The value of the integral is $-\frac{1}{3}$. The correct option is **(d)**.

CUET 2023 PYQ 36

The value of $\int_{-3}^{2} x^2|2x|\,dx$ is:

www.cuet.pw www.cbse.page

(a) 65 (b) $-\frac{65}{2}$ (c) 97 (d) $\frac{97}{2}$

Concept Builder

www.cuet.pw www.cbse.page

Step-by-Step Detailed Solution

- **Understanding the Function and Absolute Value**
  - The integrand involves the absolute value function $|2x|$.
  - Recall the definition of the modulus function:

$$|X| = \begin{cases} X & \text{if } X \geq 0 \\ -X & \text{if } X < 0 \end{cases}$$

  - Applying this to $|2x|$:

$$|2x| = \begin{cases} 2x & \text{for } x \geq 0 \\ -2x & \text{for } x < 0 \end{cases}$$

  - Therefore, the function inside the integral, $f(x) = x^2|2x|$, behaves differently for positive and negative values of x:
    - For $x < 0$: $f(x) = x^2(-2x) = -2x^3$
    - For $x \geq 0$: $f(x) = x^2(2x) = 2x^3$
- **Splitting the Definite Integral**
  - The limits of integration are from -3 to 2. Since the function definition changes at $x = 0$, we must split the integral at this point.

$$I = \int_{-3}^{2} x^2|2x|\,dx = \int_{-3}^{0} x^2|2x|\,dx + \int_{0}^{2} x^2|2x|\,dx$$

▶ Evaluating the First Integral (Interval $[-3, 0]$)

✅ In the range $-3 \le x \le 0$, $|2x| = -2x$.

$$I_1 = \int_{-3}^{0} x^2 \, (-2x) \, dx = \int_{-3}^{0} -2x^3 \, dx$$

✅ Take the constant -2 outside:

$$I_1 = -2 \int_{-3}^{0} x^3 \, dx$$

✅ Apply the power rule of integration $\int x^n \, dx = \frac{x^{n+1}}{n+1}$:

$$I_1 = -2 \left[\frac{x^4}{4} \right]_{-3}^{0}$$

✅ Simplify constants:

$$I_1 = -\frac{2}{4} \left[x^4 \right]_{-3}^{0} = -\frac{1}{2} \left(0^4 - (-3)^4 \right)$$

✅ Calculate values:

$$I_1 = -\frac{1}{2} (0 - 81) = -\frac{1}{2} (-81) = \frac{81}{2}$$

▶ Evaluating the Second Integral (Interval $[0, 2]$)

✅ In the range $0 \le x \le 2$, $|2x| = 2x$.

$$I_2 = \int_{0}^{2} x^2 \, (2x) \, dx = \int_{0}^{2} 2x^3 \, dx$$

✅ Integrate:

$$I_2 = 2 \left[\frac{x^4}{4} \right]_{0}^{2}$$

✅ Simplify and apply limits:

$$I_2 = \frac{1}{2} \left(2^4 - 0^4 \right) = \frac{1}{2} (16) = 8$$

✅ To make addition easier later, write 8 as a fraction with denominator 2:

$$I_2 = \frac{16}{2}$$

- **Calculating the Final Result**
  - Total Integral $I = I_1 + I_2$:

$$I = \frac{81}{2} + \frac{16}{2}$$

$$I = \frac{81 + 16}{2}$$

$$I = \frac{97}{2}$$

www.cuet.pw www.cbse.page

Key Concepts & Observation

- **Check for Even Function:**
  - Let $f(x) = x^2|2x|$.
  - $f(-x) = (-x)^2|2(-x)| = x^2|-2x| = x^2|2x| = f(x)$.
  - Since $f(x)$ is an even function, the graph is symmetric about the y-axis. The area is always positive.
  - This confirms that $\int_{-3}^{0} f(x)\,dx$ must be positive (which we found as $\frac{81}{2}$).
  - If the limits were symmetric (e.g., -2 to 2), we could have used $2\int_{0}^{2} f(x)dx$. Since they are not, splitting is the standard method.

www.cuet.pw www.cbse.page

Final Conclusion

- The value of the integral is $\frac{97}{2}$. The correct option is **(d)**.

CUET 2023 PYQ 37

The value of $\int \frac{dx}{x^2-6x+13}$ is:

www.cuet.pw www.cbse.page

(a) $\frac{1}{2}\tan^{-1}\frac{x-3}{2}+C$

(b) $\frac{1}{2}\cot^{-1}\frac{x-3}{2}+C$

(c) $\frac{1}{2}\tan^{-1}\frac{x+3}{2}+C$

(d) $\frac{1}{2}\cot^{-1}\frac{x+3}{2}+C$

Concept Builder

www.cuet.pw www.cbse.page

Understanding the Integral Form

The given integral is:

$$I = \int \frac{dx}{x^2-6x+13}$$

The denominator is a quadratic polynomial ax^2+bx+c where $b^2-4ac<0$ (discriminant is negative: $36-52=-16$).

For such integrals, the standard method is to **Complete the Square** in the denominator to bring it to the standard form $\int \frac{dx}{x^2+a^2}$.

Step 1: Completing the Square for the Denominator

Consider the expression:

$$x^2-6x+13$$

To complete the square, take the coefficient of x (which is -6), divide it by 2, and square it:

$$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$$

Add and subtract this value inside the expression, or simply split the constant term 13:

$$x^2-6x+13 = x^2-6x+9+4$$

- Now, group the first three terms as a perfect square:

$$(x^2 - 6x + 9) + 4 = (x - 3)^2 + 2^2$$

- **Step 2: Rewrite the Integral**
- Substitute the modified denominator back into the integral:

$$I = \int \frac{dx}{(x-3)^2 + 2^2}$$

- **Step 3: Apply the Standard Integration Formula**
- Recall the standard formula:

$$\int \frac{dx}{X^2 + A^2} = \frac{1}{A}\tan^{-1}\left(\frac{X}{A}\right) + C$$

- Comparing our integral with the standard form:
  - Variable part $X = x - 3$ (since $dX = dx$)
  - Constant part $A = 2$
- Apply the formula:

$$I = \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$$

www.cuet.pw www.cbse.page

Key Concepts & Formulae Used

- **Completing the Square:** A quadratic $x^2 + bx + c$ can be written as $\left(x + \frac{b}{2}\right)^2 + \left(c - \frac{b^2}{4}\right)$.
- **Standard Integral:**

$$\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

www.cuet.pw www.cbse.page

Final Conclusion

- The value of the integral is $\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$.
- The correct option is **(a)**.

CUET 2023 PYQ 38

The value of the integral $\int_2^4 \frac{x}{x^2+1}\,dx$ is :

www.cuet.pw www.cbse.page

(1) $\frac{1}{2}\log\left(\frac{5}{17}\right)$ (2) $\frac{1}{2}\log\left(\frac{17}{5}\right)$ (3) $2\log\left(\frac{5}{17}\right)$ (4) $2\log\left(\frac{17}{5}\right)$

Concept Builder

www.cuet.pw www.cbse.page

- **Integration by Substitution:** This method is used when the integrand contains a function and its derivative (or a scalar multiple of it).
  - **Form:** $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$
  - In definite integrals, when we change the variable of integration (e.g., from x to t), we must also change the upper and lower limits of integration to correspond to the new variable. This avoids the need to substitute back to x at the end.
- **Logarithmic Properties:** The solution involves simplifying logarithms. Key properties used are:
  - Fundamental Theorem of Calculus: $[F(x)]_a^b = F(b) - F(a)$
  - Log quotient rule: $\log a - \log b = \log\left(\frac{a}{b}\right)$

www.cuet.pw www.cbse.page

Step-by-Step Solution

- **Step 1: Analyze the Integrand** Let $I = \int_2^4 \frac{x}{x^2+1}\,dx$. Notice that the derivative of the denominator (x^2+1) is $2x$, which is very similar to the numerator (x). This suggests the method of substitution.
- **Step 2: Apply Substitution** Let $t = x^2 + 1$. Differentiating with respect to x:

$$\frac{dt}{dx} = 2x \implies dt = 2x\,dx \implies \frac{dt}{2} = x\,dx$$

- **Step 3: Change the Limits** We must convert the x-limits to t-limits using $t = x^2 + 1$.
  - **Lower Limit:** When $x = 2$, $t = (2)^2 + 1 = 4 + 1 = 5$.
  - **Upper Limit:** When $x = 4$, $t = (4)^2 + 1 = 16 + 1 = 17$.
- **Step 4: Rewrite the Integral in terms of t** Substitute $x\,dx = \dfrac{dt}{2}$ and the new limits into the integral:

$$I = \int_5^{17} \frac{1}{t} \cdot \frac{dt}{2}$$

$$I = \frac{1}{2} \int_5^{17} \frac{1}{t}\, dt$$

- **Step 5: Evaluate the Integral** The integral of $\dfrac{1}{t}$ is $\log|t|$.

$$I = \frac{1}{2} \left[\log t\right]_5^{17}$$

Apply the limits (Upper limit - Lower limit):

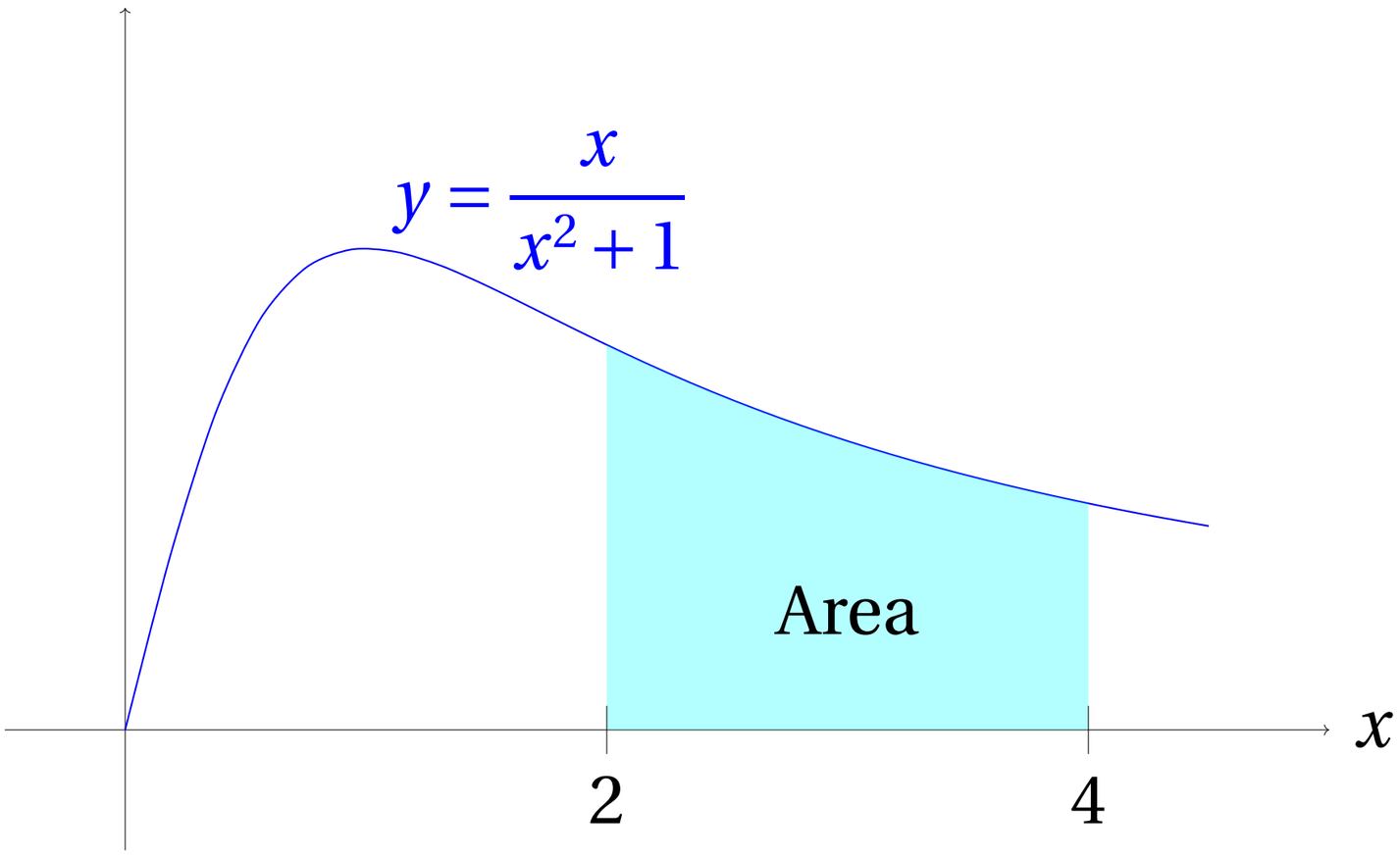
$$I = \frac{1}{2} \left(\log 17 - \log 5\right)$$

- **Step 6: Simplify using Log Properties** Using $\log a - \log b = \log\left(\dfrac{a}{b}\right)$:

$$I = \frac{1}{2} \log\left(\frac{17}{5}\right)$$

- **Conclusion:** The calculated value matches option (2).

www.cuet.pw www.cbse.page



Alternative Method (Short Trick / Inspection)

- **Direct Formula Recognition:** If you recognize the pattern $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)|$, you can solve this in one line.
- **Procedure:** Multiply and divide the integral by 2 to create the exact derivative in the numerator:

$$I = \frac{1}{2}\int_2^4 \frac{2x}{x^2+1}\,dx$$

Here, $f(x) = x^2+1$ and $f'(x) = 2x$.

$$I = \frac{1}{2}\left[\log\left(x^2+1\right)\right]_2^4$$

$$I = \frac{1}{2}\left(\log(17) - \log(5)\right) = \frac{1}{2}\log\left(\frac{17}{5}\right)$$

CUET 2023 PYQ 39

The value of $\int_0^3 |2x-6|\,dx$ is :

www.cuet.pw www.cbse.page

(1) $-\frac{9}{4}$

(2) $\frac{19}{2}$

(3) 9

(4) -3

Concept Builder

www.cuet.pw www.cbse.page

To solve definite integrals involving modulus functions (absolute value), we must use the property of splitting the limit at the point where the function inside the modulus becomes zero.

Definition of Modulus Function

The function $|f(x)|$ is defined as:

$$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$$

Steps to Integrate Modulus Functions

Step 1: Find the "critical point" by setting the expression inside the modulus to zero ($f(x) = 0$).

Step 2: Check the sign of $f(x)$ in the intervals created by this critical point.

Step 3: Split the integral $\int_a^b$ at the critical point c (if $a < c < b$) using the property:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Step-by-Step Solution

Step 1: Find the Critical Point

- Set the expression inside the modulus to zero:

$$2x - 6 = 0$$

$$2x = 6$$

$$x = 3$$

- The critical point is at $x = 3$, which happens to be the upper limit of our integral.

Step 2: Determine the Sign in the Interval

- The limits of integration are from 0 to 3.
- Pick a test point inside the interval $[0,3]$, for example, $x = 1$.

$$2(1) - 6 = 2 - 6 = -4$$

- Since the value is negative, for the interval $0 \leq x \leq 3$, we have:

$$|2x - 6| = -(2x - 6) = 6 - 2x$$

Step 3: Evaluate the Integral

- Substitute the function into the integral:

$$I = \int_0^3 (6 - 2x)\, dx$$

- Integrate term by term:

$$I = \left[6x - \frac{2x^2}{2} \right]_0^3$$

$$I = \left[6x - x^2 \right]_0^3$$

- Apply limits (Upper Limit - Lower Limit):

$$I = \left(6(3) - (3)^2\right) - \left(6(0) - 0^2\right)$$

$$I = (18 - 9) - 0$$

$$I = 9$$

www.cuet.pw www.cbse.page

Alternative Method: Graphical / Geometric (Short Trick)

- **Area under the curve interpretation**

✅ The graph of $y = |2x - 6|$ represents a "V" shape with its vertex touching the x-axis at $x = 3$.

✅ The definite integral $\int_0^3 |2x - 6|\, dx$ represents the area of the region bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 3$.

✅ This region forms a right-angled triangle:

✅ **Base:** From $x = 0$ to $x = 3 \implies$ Base $= 3$.

✅ **Height:** The value of the function at $x = 0$.

$$y = |2(0) - 6| = |-6| = 6$$

So, Height $= 6$.

✅ **Area of Triangle:**

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$\text{Area} = \frac{1}{2} \times 3 \times 6$$

$$\text{Area} = 3 \times 3 = 9$$

✅ This confirms the answer is 9 without needing integration.

www.cuet.pw www.cbse.page

The value of the integral is 9**.** Comparing with the given options:

(1) $-\frac{9}{4}$

(2) $\frac{19}{2}$

(3) 9 ✅ **(Correct Option)**

(4) -3

CUET 2023 PYQ 40

The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals :

www.cuet.pw www.cbse.page

(1) $\left(\frac{x^4+1}{x^4}\right)^{1/4}+C$

(2) $\left(x^4+1\right)^{1/4}+C$

(3) $-\left(x^4+1\right)^{1/4}+C$

(4) $-\left(\frac{x^4+1}{x^4}\right)^{1/4}+C$

Concept Builder

www.cuet.pw www.cbse.page

To solve integrals involving fractional powers of polynomials like $(x^n+1)^{1/n}$ or similar forms, a standard algebraic manipulation trick is very effective.

- **The Factoring Trick**
  - When you have a term like $(x^n+a)^{m/n}$ in the denominator, factor out the highest power of x (i.e., x^n) from inside the bracket.
  - This operation transforms the term inside the bracket into $(1+ax^{-n})$.
  - The derivative of $(1+ax^{-n})$ will involve $x^{-(n+1)}$, which often matches the term remaining outside the bracket in the numerator (or can be formed easily).
  - This sets up a perfect substitution method.
- **Power Rule for Integration**
  - $\int x^n dx=\frac{x^{n+1}}{n+1}+C$ (where $n\neq -1$).

www.cuet.pw www.cbse.page

Step-by-Step Solution

▶ Step 1: Simplify the Denominator

✅ The integral is $I = \int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$.

✅ Factor out x^4 from the term inside the bracket $\left(x^4+1\right)$:

$$\left(x^4+1\right)^{3/4} = \left[x^4\left(1+\frac{1}{x^4}\right)\right]^{3/4}$$

✅ Apply the power laws $(ab)^n = a^n b^n$:

$$= \left(x^4\right)^{3/4}\left(1+\frac{1}{x^4}\right)^{3/4}$$

$$= x^3\left(1+x^{-4}\right)^{3/4}$$

▶ Step 2: Rewrite the Integral

✅ Substitute the simplified term back into the integral:

$$I = \int \frac{dx}{x^2 \cdot x^3\left(1+x^{-4}\right)^{3/4}}$$

$$I = \int \frac{dx}{x^5\left(1+x^{-4}\right)^{3/4}}$$

✅ Bring x^5 to the numerator as x^{-5}:

$$I = \int \frac{x^{-5}dx}{\left(1+x^{-4}\right)^{3/4}}$$

▶ Step 3: Apply Substitution Method

✅ Let $u = 1+x^{-4}$.

✅ Differentiate with respect to x:

$$du = -4x^{-5}dx$$

$$\implies x^{-5}dx = -\frac{1}{4}du$$

▶ Step 4: Solve the Integral in terms of u

✅ Substitute u and du into the integral:

$$I = \int \frac{-\frac{1}{4}du}{u^{3/4}}$$

$$I = -\frac{1}{4}\int u^{-3/4}du$$

Apply the power rule $\int u^n du = \frac{u^{n+1}}{n+1}$:

$$I = -\frac{1}{4}\left[\frac{u^{-3/4+1}}{-\frac{3}{4}+1}\right] + C$$

$$I = -\frac{1}{4}\left[\frac{u^{1/4}}{\frac{1}{4}}\right] + C$$

$$I = -\frac{1}{4} \cdot 4 \cdot u^{1/4} + C$$

$$I = -u^{1/4} + C$$

Step 5: Back-Substitution

Replace u with $1 + \frac{1}{x^4}$:

$$I = -\left(1 + \frac{1}{x^4}\right)^{1/4} + C$$

$$I = -\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$$

www.cuet.pw www.cbse.page

Final Answer : The value of the integral is $-\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$.

Comparing with the given options:

(1) $\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$

(2) $\left(x^4+1\right)^{1/4} + C$

(3) $-\left(x^4+1\right)^{1/4} + C$

(4) $-\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$ **(Correct Option)**

CUET 2023 PYQ 41

The integral $\int_0^1 x(1-x)^n\,dx$ is equal to :

www.cuet.pw www.cbse.page

(1) $\dfrac{1}{(n+2)(n+3)}$

(2) $\dfrac{1}{(n+1)(n+2)}$

(3) $\dfrac{1}{n(n+1)}$

(4) $\dfrac{1}{(n-1)(n-2)}$

Concept Builder

www.cuet.pw www.cbse.page

To solve definite integrals of the form $\int_0^a f(x)\,dx$, one of the most powerful tools is the property of "Reflection" or "King's Rule".

- **Property of Definite Integrals (King's Rule)**
  - The property states:

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

  - This property is particularly useful when the integrand contains terms like $(a-x)^n$, as it simplifies the expression by shifting the complex power to a single variable.
- **Power Rule of Integration**
  - The standard integral formula:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (\text{for } n \neq -1)$$

www.cuet.pw www.cbse.page

Step-by-Step Solution

Let the given integral be I.

$$I = \int_0^1 x\,(1-x)^n\,dx$$

- **Step 1: Apply the Property** $\int_0^a f\,(x)\,dx = \int_0^a f\,(a-x)\,dx$

  - Here, the upper limit $a = 1$.
  - Replace x with $(1-x)$ in the integrand.

$$I = \int_0^1 (1-x)\,[1-(1-x)]^n\,dx$$

- **Step 2: Simplify the Integrand**

  - Simplify the term inside the bracket: $[1-1+x]^n = x^n$.

$$I = \int_0^1 (1-x)\,x^n\,dx$$

  - Now, distribute x^n inside the bracket:

$$I = \int_0^1 \left(x^n - x\cdot x^n\right)dx$$

$$I = \int_0^1 \left(x^n - x^{n+1}\right)dx$$

- **Step 3: Integrate Term by Term**

  - Apply the power rule $\int x^k\,dx = \frac{x^{k+1}}{k+1}$:

$$I = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

- **Step 4: Apply Limits**

  - Substitute the upper limit ($x = 1$) and lower limit ($x = 0$):

$$I = \left(\frac{1^{n+1}}{n+1} - \frac{1^{n+2}}{n+2}\right) - \left(\frac{0}{n+1} - \frac{0}{n+2}\right)$$

$$I = \frac{1}{n+1} - \frac{1}{n+2}$$

▶ Step 5: Simplify the Final Fraction

✅ Take the LCM of the denominators:

$$I = \frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$I = \frac{n+2-n-1}{(n+1)(n+2)}$$

$$I = \frac{1}{(n+1)(n+2)}$$

www.cuet.pw www.cbse.page

Alternative Method (Beta Function - Short Trick)

For competitive exams, this integral is a direct form of the Beta Function.

▶ Beta Function Formula

✅ The Beta function is defined as:

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx = \frac{(m-1)!\,(n-1)!}{(m+n-1)!}$$

✅ In our integral $\int_0^1 x^1(1-x)^n\,dx$:

✅ Comparing powers: $m-1=1 \Rightarrow m=2$.

✅ Comparing powers: $n-1=n \Rightarrow$ parameter is $n+1$.

✅ Using the factorial formula:

$$I = \frac{1!\cdot n!}{(2+n)!}$$

$$I = \frac{1\cdot n!}{(n+2)(n+1)\,n!}$$

$$I = \frac{1}{(n+1)(n+2)}$$

www.cuet.pw www.cbse.page

Final Answer : ✅ **(Correct Option)** is (2) $\frac{1}{(n+1)(n+2)}$.

CUET 2023 PYQ 42

Match List I with List II

www.cuet.pw www.cbse.page

LIST I

(A) $\int \frac{\sin x}{1+\cos x}\,dx$

(B) $\int \frac{1}{1-\tan x}\,dx$

(C) $\int \frac{e^{\tan^{-1} x}}{1+x^2}\,dx$

(D) $\int \frac{1}{x+x\log x}\,dx$

www.cuet.pw www.cbse.page

LIST II

(I) $e^{\tan^{-1} x}+C$

(II) $\log(\log x+1)+C$

(III) $-\log|1+\cos x|+C$

(IV) $\frac{x}{2}-\frac{1}{2}\log|\cos x-\sin x|+C$

www.cuet.pw www.cbse.page

Choose the correct answer from the options given below:

(1) A-II, B-III, C-IV, D-I

(2) A-III, B-IV, C-I, D-II

(3) A-I, B-II, C-III, D-IV

(4) A-IV, B-I, C-III, D-II

Concept Builder

www.cuet.pw www.cbse.page

Detailed Explanation and Concepts

- **Part A: Solving** $\int \frac{\sin x}{1+\cos x}\,dx$
  - **Method: Substitution** Let $1+\cos x = t$. Differentiating with respect to x:

$$-\sin x\,dx = dt \implies \sin x\,dx = -dt$$

  - Substitute into the integral:

$$I = \int \frac{-dt}{t} = -\int \frac{1}{t}\,dt$$

$$I = -\log|t| + C$$

  - Put back $t = 1+\cos x$:

$$I = -\log|1+\cos x| + C$$

  - **Match:** A matches with **III.**

- **Part C: Solving** $\int \frac{e^{\tan^{-1} x}}{1+x^2}\,dx$
  - **Method: Substitution** Let $\tan^{-1} x = t$. Differentiating with respect to x:

$$\frac{1}{1+x^2}\,dx = dt$$

  - Substitute into the integral:

$$I = \int e^t\,dt$$

$$I = e^t + C$$

  - Put back $t = \tan^{-1} x$:

$$I = e^{\tan^{-1} x} + C$$

  - **Match:** C matches with **I.**

- **Part D: Solving** $\int \frac{1}{x+x\log x}\,dx$
  - **Simplification:** Take x common in the denominator:

$$I = \int \frac{1}{x(1+\log x)}\,dx$$

- **Method: Substitution** Let $1+\log x = t$. Differentiating:

$$\frac{1}{x}dx = dt$$

- Substitute into the integral:

$$I = \int \frac{1}{t}dt$$

$$I = \log|t| + C$$

- Put back $t = 1+\log x$:

$$I = \log|1+\log x| + C$$

- **Match:** D matches with **II**.

▶ **Part B: Solving** $\int \frac{1}{1-\tan x}dx$

- **Simplification:** Convert $\tan x$ to $\frac{\sin x}{\cos x}$:

$$I = \int \frac{1}{1-\frac{\sin x}{\cos x}}dx = \int \frac{\cos x}{\cos x - \sin x}dx$$

- **Standard Manipulation Step:** Multiply and divide by 2:

$$I = \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx$$

Rewrite $2\cos x$ as $(\cos x - \sin x) + (\cos x + \sin x)$:

$$I = \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{\cos x - \sin x}dx$$

Split the integral:

$$I = \frac{1}{2}\int \left(1 + \frac{\cos x + \sin x}{\cos x - \sin x}\right)dx$$

- **Integration:** First part: $\int 1\,dx = x$. Second part: Let $\cos x - \sin x = u$, then $(-\sin x - \cos x)dx = du \implies (\sin x + \cos x)dx = -du$.

$$\int \frac{-du}{u} = -\log|u| = -\log|\cos x - \sin x|$$

- **Final Result:**

$$I = \frac{1}{2}x - \frac{1}{2}\log|\cos x - \sin x| + C$$

- **Match:** B matches with **IV**.

- **Conclusion**
  - The matches are: A → III, B → IV, C → I, D → II.
  - This corresponds to Option **2**.

www.cuet.pw www.cbse.page

Short Trick / Strategic Solving

- **Identify the Easiest Integrals First**
  - **Item C** involves $e^{\tan^{-1} x}$ and its derivative $\frac{1}{1+x^2}$. This is a direct substitution pattern $\int e^{f(x)} f'(x)\,dx = e^{f(x)}$.
  - Immediately, C must match with **I** ($e^{\tan^{-1} x}$).
  - **Item A** has $\frac{\sin x}{1+\cos x}$. The numerator is the derivative of the denominator (with a negative sign). The result must involve a log. Since there is a negative sign, it matches **III** ($-\log|1+\cos x|$).
  - Now check the options:
    * Option 1: C-IV (Incorrect)
    * Option 2: C-I (Correct)
    * Option 3: C-III (Incorrect)
    * Option 4: C-III (Incorrect)
  - By solving just **Item C**, you can eliminate all other options and select Option **2** instantly.

CUET 2023 PYQ 43

$$\int \left(\frac{1+x+x^2}{1+x^2}\right) e^{\tan^{-1}x}\, dx =$$

www.cuet.pw www.cbse.page

(1) $x + e^{\tan^{-1}x} + c$

(2) $e^{\tan^{-1}x} - x + c$

(3) $e^{\tan^{-1}x} + c$

(4) $xe^{\tan^{-1}x} + c$

Concept Builder

www.cuet.pw www.cbse.page

Detailed Explanation and Concepts

- **Key Concept: Integration by Parts**
  - Formula: $\int u\,dv = uv - \int v\,du.$
  - This method is used when the integrand involves a product of two different types of functions (e.g., algebraic and exponential).
- **Step-by-Step Solution (Method 1: Splitting the Integral)**
  - **Step 1: Simplify the Integrand** Split the numerator to separate the $1 + x^2$ term:

$$I = \int \left(\frac{(1+x^2)+x}{1+x^2}\right) e^{\tan^{-1}x}\, dx$$

$$I = \int \left(1 + \frac{x}{1+x^2}\right) e^{\tan^{-1}x}\, dx$$

  - **Step 2: Separate into Two Integrals**

$$I = \int e^{\tan^{-1}x}\, dx + \int \frac{xe^{\tan^{-1}x}}{1+x^2}\, dx$$

Let's evaluate the first integral, $\int e^{\tan^{-1}x}\, dx$, using Integration by Parts, and leave the second integral as is.

Step 3: Apply Integration by Parts For $\int 1 \cdot e^{\tan^{-1} x}\, dx$:

* Let $u = e^{\tan^{-1} x} \implies du = e^{\tan^{-1} x} \cdot \dfrac{1}{1+x^2}\, dx$
* Let $dv = 1\, dx \implies v = x$

Applying the formula ($uv - \int v\, du$):

$$\int e^{\tan^{-1} x}\, dx = x e^{\tan^{-1} x} - \int x \cdot \frac{e^{\tan^{-1} x}}{1+x^2}\, dx$$

Step 4: Combine Results Substitute this back into the equation for I:

$$I = \left(x e^{\tan^{-1} x} - \int \frac{x e^{\tan^{-1} x}}{1+x^2}\, dx \right) + \int \frac{x e^{\tan^{-1} x}}{1+x^2}\, dx$$

The two integral terms are identical with opposite signs, so they cancel out:

$$I = x e^{\tan^{-1} x} + C$$

Thus, Option **4** is correct.

www.cuet.pw www.cbse.page

Alternative Method (Substitution)

Using Substitution to find the Standard Form

Let $t = \tan^{-1} x$.

Then $x = \tan t$, and differentiating gives $dx = \sec^2 t\, dt$.

Also, note that $1 + x^2 = 1 + \tan^2 t = \sec^2 t$.

Substitute these into the integral:

$$I = \int \left(\frac{1 + \tan t + \tan^2 t}{\sec^2 t} \right) e^t (\sec^2 t\, dt)$$

Cancel $\sec^2 t$:

$$I = \int (1 + \tan^2 t + \tan t) e^t\, dt$$

Using the identity $1 + \tan^2 t = \sec^2 t$:

$$I = \int (\sec^2 t + \tan t) e^t\, dt$$

$$I = \int e^t (\tan t + \sec^2 t)\, dt$$

- **Concept:** This matches the standard form $\int e^x (f(x) + f'(x))\,dx = e^x f(x) + C$. Here, $f(t) = \tan t$ and $f'(t) = \sec^2 t$.
- **Result:**

$$I = e^t \tan t + C$$

Substitute back $t = \tan^{-1} x$ (so $\tan t = x$):

$$I = x e^{\tan^{-1} x} + C$$

www.cuet.pw www.cbse.page

Short Trick / Reverse Method

- **Differentiation of Options**
  - In integration problems, differentiation is the reverse process. You can differentiate the options to see which one yields the integrand.
  - Check Option 4: $y = x e^{\tan^{-1} x}$. Apply Product Rule:

$$\frac{dy}{dx} = 1 \cdot e^{\tan^{-1} x} + x \cdot \frac{d}{dx}\left(e^{\tan^{-1} x}\right)$$

$$\frac{dy}{dx} = e^{\tan^{-1} x} + x \cdot e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = e^{\tan^{-1} x}\left(1 + \frac{x}{1+x^2}\right)$$

$$\frac{dy}{dx} = e^{\tan^{-1} x}\left(\frac{1+x^2+x}{1+x^2}\right)$$

  - This exactly matches the integrand given in the question.

CUET 2022 PYQ 44

If $y = \left(\frac{1}{x}\right)^x$, then value of $e^e \left(\frac{d^2y}{dx^2}\right)_{x=e}$ is:

www.cuet.pw www.cbse.page

(a) $2 - \frac{1}{e}$ (b) $4 - \frac{1}{e}$ (c) $\frac{1}{e}$ (d) $1 - \frac{1}{e}$

Concept Builder

www.cuet.pw www.cbse.page

Goal: Our objective is to find the value of the expression

$$e^e \left(\frac{d^2y}{dx^2}\right)_{x=e}$$

given the function $y = \left(\frac{1}{x}\right)^x$

www.cuet.pw www.cbse.page

Step 1: Simplify the function and apply logarithmic differentiation

First, let's simplify the expression for y:

$$y = \left(\frac{1}{x}\right)^x = (x^{-1})^x = x^{-x}$$

Since the function involves a variable in both the base and the exponent, the easiest way to differentiate it is by using logarithmic differentiation.

www.cuet.pw www.cbse.page

Take the natural logarithm (ln) of both sides:

$$\ln y = \ln\left(x^{-x}\right)$$

www.cuet.pw www.cbse.page

Using the logarithm power rule, $\ln\left(a^b\right) = b \ln a$, we get:

$$\ln y = -x \ln x$$

www.cuet.pw www.cbse.page

Step 2: Find the first derivative ($\frac{dy}{dx}$)

Now, we differentiate both sides of the equation $\ln y = -x \ln x$ with respect to x. For the left side, $\frac{d}{dx}(\ln y)$, we use implicit differentiation:

$$\frac{d}{dx}(\ln y) = \frac{1}{y} \cdot \frac{dy}{dx}$$

www.cuet.pw www.cbse.page

For the right side, $\frac{d}{dx}(-x \ln x)$, we use the product rule, where $\frac{d}{dx}(uv) = u'v + uv'$. Let $u = -x$ and $v = \ln x$. Then $u' = -1$ and $v' = \frac{1}{x}$.

www.cuet.pw www.cbse.page

$$\frac{d}{dx}(-x \ln x) = (-1)(\ln x) + (-x)\left(\frac{1}{x}\right) = -\ln x - 1 = -(\ln x + 1)$$

www.cuet.pw www.cbse.page

Now, we set the differentiated left and right sides equal to each other:

$$\frac{1}{y}\frac{dy}{dx} = -(\ln x + 1)$$

To solve for $\frac{dy}{dx}$, multiply both sides by y:

$$\frac{dy}{dx} = -y(\ln x + 1)$$

www.cuet.pw www.cbse.page

Finally, substitute the original expression for y (which is $y = x^{-x}$) back into the equation:

$$\frac{dy}{dx} = -x^{-x}(\ln x + 1)$$

www.cuet.pw www.cbse.page

Step 3: Find the second derivative ($\frac{d^2y}{dx^2}$)

To find the second derivative, $\frac{d^2y}{dx^2}$, we must differentiate the first derivative, $\frac{dy}{dx} = -x^{-x}(\ln x + 1)$, with respect to x. We will use the product rule again.

www.cuet.pw www.cbse.page

Let $u = -x^{-x}$ and $v = (\ln x + 1)$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(uv) = u'v + uv'$$

We need to find u' and v':

- **Find u':** $u' = \frac{d}{dx}(-x^{-x}) = -\frac{d}{dx}(x^{-x})$.
- From Step 2, we already know that $\frac{d}{dx}(x^{-x}) = \frac{dy}{dx} = -x^{-x}(\ln x + 1)$.
- Therefore, $u' = -[-x^{-x}(\ln x + 1)] = x^{-x}(\ln x + 1)$.
- **Find v':** $v' = \frac{d}{dx}(\ln x + 1) = \frac{1}{x}$.

www.cuet.pw www.cbse.page

Now, substitute u, v, u', and v' back into the product rule formula:

$$\frac{d^2y}{dx^2} = \left[x^{-x}(\ln x + 1)\right](\ln x + 1) + (-x^{-x})\left(\frac{1}{x}\right)$$

Simplify the expression:

$$\frac{d^2y}{dx^2} = x^{-x}(\ln x + 1)^2 - \frac{x^{-x}}{x}$$

www.cuet.pw www.cbse.page

We can rewrite $\frac{x^{-x}}{x}$ as $x^{-x} \cdot x^{-1} = x^{-(x+1)}$.

$$\frac{d^2y}{dx^2} = x^{-x}(\ln x + 1)^2 - x^{-(x+1)}$$

www.cuet.pw www.cbse.page

For easier evaluation, let's factor out x^{-x}:

$$\frac{d^2y}{dx^2} = x^{-x}\left[(\ln x + 1)^2 - \frac{1}{x}\right]$$

www.cuet.pw www.cbse.page

Step 4: Evaluate the second derivative at $x = e$

Now we substitute $x = e$ into our expression for $\frac{d^2y}{dx^2}$:

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = e^{-e}\left[(\ln e + 1)^2 - \frac{1}{e}\right]$$

We know that $\ln e = 1$. Let's substitute this value in:

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = e^{-e}\left[(1+1)^2 - \frac{1}{e}\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = e^{-e}\left[(2)^2 - \frac{1}{e}\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = e^{-e}\left[4 - \frac{1}{e}\right]$$

www.cuet.pw www.cbse.page

Step 5: Calculate the final required value

The question asks for the value of $e^e\left(\frac{d^2y}{dx^2}\right)_{x=e}$. We multiply our result from Step 4 by e^e:

www.cuet.pw www.cbse.page

$$\text{Value} = e^e \times \left[e^{-e}\left(4 - \frac{1}{e}\right)\right]$$

www.cuet.pw www.cbse.page

Group the exponential terms:

$$\text{Value} = (e^e \cdot e^{-e})\left(4 - \frac{1}{e}\right)$$

www.cuet.pw www.cbse.page

Using the exponent rule $a^m \cdot a^n = a^{m+n}$:

$$e^e \cdot e^{-e} = e^{e-e} = e^0 = 1$$

www.cuet.pw www.cbse.page

Therefore, the expression simplifies to:

$$\text{Value} = 1 \cdot \left(4 - \frac{1}{e}\right) = 4 - \frac{1}{e}$$

www.cuet.pw www.cbse.page

Conclusion : The final value is $4 - \frac{1}{e}$. This matches option B (which is written as $4\frac{1}{e}$, a common typographical representation of $4 - \frac{1}{e}$ in this context). The correct option is **B**.

CUET 2022 PYQ 45

$$\int \frac{x^2+1}{x^4+1}\,dx =$$

www.cuet.pw www.cbse.page

(A) $\log\left(x^4+1\right)+C$

(B) $\log\left(\frac{x^2-1}{x^2+1}\right)+C$

(C) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+C$

(D) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)+C$

Concept Builder

www.cuet.pw www.cbse.page

Step-by-Step Solution

- Given the integral:

$$I=\int \frac{x^2+1}{x^4+1}\,dx$$

- **Step 1: Divide Numerator and Denominator by** x^2 To simplify the expression, divide every term by x^2:

$$I=\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$$

- **Step 2: Manipulate the Denominator** We want the denominator to express a perfect square involving a term whose derivative is in the numerator.
  - The numerator is $\left(1+\frac{1}{x^2}\right)$.

- Recall that $\frac{d}{dx}\left(x-\frac{1}{x}\right)=1-\left(-\frac{1}{x^2}\right)=1+\frac{1}{x^2}$.
- Therefore, we should try to express the denominator in terms of $\left(x-\frac{1}{x}\right)$.

- Use the algebraic identity: $a^2+b^2=(a-b)^2+2ab$. Here $a=x$ and $b=1/x$, so:

$$x^2+\frac{1}{x^2}=\left(x-\frac{1}{x}\right)^2+2$$

- Substitute this back into the integral:

$$I=\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}\,dx$$

- **Step 3: Substitution Method** Let $t=x-\frac{1}{x}$. Differentiating with respect to x:

$$dt=\left(1+\frac{1}{x^2}\right)dx$$

- Now the integral becomes:

$$I=\int \frac{dt}{t^2+2}$$

$$I=\int \frac{dt}{t^2+(\sqrt{2})^2}$$

- **Step 4: Use Standard Integral Formula** Using the formula $\int \frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$: Here, $a=\sqrt{2}$.

$$I=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)+C$$

- **Step 5: Back-Substitution** Replace t with $x-\frac{1}{x}=\frac{x^2-1}{x}$:

$$I=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\frac{x^2-1}{x}}{\sqrt{2}}\right)+C$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + C$$

- **Conclusion:** This matches Option (C).

www.cuet.pw www.cbse.page

Teaching Guide & Key Concepts

The "Twin Integrals" Form

- This is a classic special integral type often seen in exams.
- **Form 1:** $\int \frac{x^2+1}{x^4+1}\,dx$ or $\int \frac{x^2+1}{x^4+kx^2+1}\,dx$.
- **Form 2:** $\int \frac{x^2-1}{x^4+1}\,dx$.
- **Strategy:** Always start by dividing the numerator and denominator by x^2. This reveals terms like $(1+1/x^2)$ or $(1-1/x^2)$, which are derivatives of $(x-1/x)$ and $(x+1/x)$ respectively.

Completing the Square in Denominator

- If Numerator has $(+)$, convert Denominator to $(-)^2 + \text{const}$.
- If Numerator has $(-)$, convert Denominator to $(+)^2 - \text{const}$.
- Example: For $\int \frac{1+1/x^2}{x^2+1/x^2}$, we use $(x-1/x)^2+2$ in the denominator.

www.cuet.pw www.cbse.page

Short Tricks & Methods

Reverse Engineering (Differentiation)

- Since integration is the reverse of differentiation, you can differentiate the options.
- Differentiating $\tan^{-1} u$ gives $\frac{1}{1+u^2} \cdot u'$.
- Let's check Option (C): $y = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$.
- Let $u = \frac{x-1/x}{\sqrt{2}}$. Then $u' = \frac{1+1/x^2}{\sqrt{2}}$.
- $y' = \frac{1}{\sqrt{2}} \cdot \frac{1}{1+\frac{(x-1/x)^2}{2}} \cdot \frac{1+1/x^2}{\sqrt{2}}$

- $y' = \frac{1}{2} \cdot \frac{1}{\frac{2 + (x - 1/x)^2}{2}} \cdot (1 + 1/x^2) = \frac{1 + 1/x^2}{2 + x^2 + 1/x^2 - 2} = \frac{1 + 1/x^2}{x^2 + 1/x^2}.$
- Multiply by x^2/x^2: $\frac{x^2 + 1}{x^4 + 1}$. Match found!

CUET 2022 PYQ 46

If $y = \left(\frac{1}{x}\right)^x$, then value of $e^e \left(\frac{d^2y}{dx^2}\right)_{x=e}$ is:

www.cuet.pw www.cbse.page

(a) $2 - \frac{1}{e}$

(b) $4 - \frac{1}{e}$

(c) $\frac{1}{e}$

(d) $1 - \frac{1}{e}$

Concept Builder

www.cuet.pw www.cbse.page

Goal: Our objective is to find the value of the expression

$$e^e \left(\frac{d^2y}{dx^2}\right)_{x=e}$$

given the function $y = \left(\frac{1}{x}\right)^x$

www.cuet.pw www.cbse.page

Step 1: Simplify the function and apply logarithmic differentiation

First, let's simplify the expression for y:

$$y = \left(\frac{1}{x}\right)^x = (x^{-1})^x = x^{-x}$$

Since the function involves a variable in both the base and the exponent, the easiest way to differentiate it is by using logarithmic differentiation.

www.cuet.pw www.cbse.page

Take the natural logarithm (ln) of both sides:

$$\ln y = \ln\left(x^{-x}\right)$$

www.cuet.pw www.cbse.page

Using the logarithm power rule, $\ln\left(a^b\right) = b\ln a$, we get:

$$\ln y = -x\ln x$$

www.cuet.pw www.cbse.page

Step 2: Find the first derivative ($\frac{dy}{dx}$)

Now, we differentiate both sides of the equation $\ln y = -x\ln x$ with respect to x. For the left side, $\frac{d}{dx}(\ln y)$, we use implicit differentiation:

$$\frac{d}{dx}(\ln y) = \frac{1}{y}\cdot\frac{dy}{dx}$$

www.cuet.pw www.cbse.page

For the right side, $\frac{d}{dx}(-x\ln x)$, we use the product rule, where $\frac{d}{dx}(uv) = u'v + uv'$. Let $u = -x$ and $v = \ln x$. Then $u' = -1$ and $v' = \frac{1}{x}$.

www.cuet.pw www.cbse.page

$$\frac{d}{dx}(-x\ln x) = (-1)(\ln x) + (-x)\left(\frac{1}{x}\right) = -\ln x - 1 = -(\ln x + 1)$$

www.cuet.pw www.cbse.page

Now, we set the differentiated left and right sides equal to each other:

$$\frac{1}{y}\frac{dy}{dx} = -(\ln x + 1)$$

To solve for $\frac{dy}{dx}$, multiply both sides by y:

$$\frac{dy}{dx} = -y(\ln x + 1)$$

www.cuet.pw www.cbse.page

Finally, substitute the original expression for y (which is $y = x^{-x}$) back into the equation:

$$\frac{dy}{dx} = -x^{-x}(\ln x + 1)$$

www.cuet.pw www.cbse.page

Step 3: Find the second derivative ($\frac{d^2y}{dx^2}$)

To find the second derivative, $\frac{d^2y}{dx^2}$, we must differentiate the first derivative, $\frac{dy}{dx} = -x^{-x}(\ln x + 1)$, with respect to x. We will use the product rule again.

www.cuet.pw www.cbse.page

Let $u = -x^{-x}$ and $v = (\ln x + 1)$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(uv) = u'v + uv'$$

We need to find u' and v':

- **Find u':** $u' = \frac{d}{dx}(-x^{-x}) = -\frac{d}{dx}(x^{-x})$.
- From Step 2, we already know that $\frac{d}{dx}(x^{-x}) = \frac{dy}{dx} = -x^{-x}(\ln x + 1)$.
- Therefore, $u' = -[-x^{-x}(\ln x + 1)] = x^{-x}(\ln x + 1)$.
- **Find v':** $v' = \frac{d}{dx}(\ln x + 1) = \frac{1}{x}$.

www.cuet.pw www.cbse.page

Now, substitute u, v, u', and v' back into the product rule formula:

$$\frac{d^2y}{dx^2} = \left[x^{-x}(\ln x + 1)\right](\ln x + 1) + (-x^{-x})\left(\frac{1}{x}\right)$$

Simplify the expression:

$$\frac{d^2y}{dx^2} = x^{-x}(\ln x + 1)^2 - \frac{x^{-x}}{x}$$

www.cuet.pw www.cbse.page

We can rewrite $\frac{x^{-x}}{x}$ as $x^{-x} \cdot x^{-1} = x^{-(x+1)}$.

$$\frac{d^2y}{dx^2} = x^{-x}(\ln x + 1)^2 - x^{-(x+1)}$$

www.cuet.pw www.cbse.page

For easier evaluation, let's factor out x^{-x}:

$$\frac{d^2y}{dx^2} = x^{-x}\left[(\ln x + 1)^2 - \frac{1}{x}\right]$$

www.cuet.pw www.cbse.page

Step 4: Evaluate the second derivative at $x = e$

Now we substitute $x = e$ into our expression for $\frac{d^2y}{dx^2}$:

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = e^{-e}\left[(\ln e + 1)^2 - \frac{1}{e}\right]$$

We know that $\ln e = 1$. Let's substitute this value in:

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = e^{-e}\left[(1+1)^2 - \frac{1}{e}\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = e^{-e}\left[(2)^2 - \frac{1}{e}\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = e^{-e}\left[4 - \frac{1}{e}\right]$$

www.cuet.pw www.cbse.page

Step 5: Calculate the final required value

The question asks for the value of $e^e\left(\frac{d^2y}{dx^2}\right)_{x=e}$. We multiply our result from Step 4 by e^e:

www.cuet.pw www.cbse.page

$$\text{Value} = e^e \times \left[e^{-e}\left(4 - \frac{1}{e}\right)\right]$$

www.cuet.pw www.cbse.page

Group the exponential terms:

$$\text{Value} = (e^e \cdot e^{-e})\left(4 - \frac{1}{e}\right)$$

www.cuet.pw www.cbse.page

Using the exponent rule $a^m \cdot a^n = a^{m+n}$:

$$e^e \cdot e^{-e} = e^{e-e} = e^0 = 1$$

www.cuet.pw www.cbse.page

Therefore, the expression simplifies to:

$$\text{Value} = 1 \cdot \left(4 - \frac{1}{e}\right) = 4 - \frac{1}{e}$$

www.cuet.pw www.cbse.page

Conclusion : The final value is $4 - \frac{1}{e}$. This matches option B (which is written as $4\frac{1}{e}$, a common typographical representation of $4 - \frac{1}{e}$ in this context). The correct option is **B**.

CUET 2022 PYQ 47

$\int \frac{dx}{x(x^5+3)}$ is equal to :

www.cuet.pw www.cbse.page

(a) $\frac{1}{3}\log\left|\frac{x^5}{x^5+3}\right|+C$

(b) $\frac{1}{15}\log\left|\frac{x^5}{x^5+3}\right|+C$

(c) $\frac{1}{5}\log\left|\frac{x^5}{x^5+3}\right|+C$

(d) $\frac{1}{25}\log\left|\frac{x^5}{x^5+3}\right|+C$

Concept Builder

www.cuet.pw www.cbse.page

⋆ Detailed Solution and Concept Explanation

The problem requires us to evaluate the indefinite integral $I = \int \frac{dx}{x(x^5+3)}$.
This is a classic form solved using substitution and partial fractions.

www.cuet.pw www.cbse.page

1. Key Concept: Strategic Substitution

The integral is of the form $\int \frac{dx}{x(x^n+a)}$. The key step is to manipulate the integrand so that the derivative of the term in the parentheses (x^n) appears in the numerator, allowing for a standard substitution.

www.cuet.pw www.cbse.page

2. Step-by-Step Method

(1) **Manipulate the Integrand:** To make substitution easier, we multiply the numerator and denominator by $x^{n-1} = x^4$.

$$I = \int \frac{x^4\,dx}{x \cdot x^4(x^5+3)} = \int \frac{x^4\,dx}{x^5(x^5+3)}$$

www.cuet.pw www.cbse.page

(2) Apply Substitution: Let $u = x^5$. Differentiating both sides with respect to x:

$$\frac{du}{dx} = 5x^4 \implies x^4\,dx = \frac{1}{5}du$$

Substitute u into the integral:

$$I = \int \frac{\frac{1}{5}du}{u(u+3)} = \frac{1}{5}\int \frac{du}{u(u+3)}$$

www.cuet.pw www.cbse.page

(3) Resolve using Partial Fractions: We use the partial fraction decomposition for $\frac{1}{u(u+3)}$. We can rewrite the numerator 1 as $\frac{1}{3}[(u+3)-u]$:

$$\frac{1}{u(u+3)} = \frac{1}{3}\left[\frac{(u+3)-u}{u(u+3)}\right] = \frac{1}{3}\left[\frac{1}{u} - \frac{1}{u+3}\right]$$

Substitute back into I:

$$I = \frac{1}{5}\cdot\frac{1}{3}\int\left(\frac{1}{u} - \frac{1}{u+3}\right)du = \frac{1}{15}\int\left(\frac{1}{u} - \frac{1}{u+3}\right)du$$

www.cuet.pw www.cbse.page

(4) Integrate and Resubstitute:

$$I = \frac{1}{15}\left[\log|u| - \log|u+3|\right] + C$$

Using the logarithm property $\log a - \log b = \log\frac{a}{b}$:

$$I = \frac{1}{15}\log\left|\frac{u}{u+3}\right| + C$$

Substitute back $u = x^5$:

$$I = \frac{1}{15}\log\left|\frac{x^5}{x^5+3}\right| + C$$

www.cuet.pw www.cbse.page

⋆ Short Trick / General Formula

The integral $\int \frac{dx}{x(x^n + a)}$ follows a standard formula derived from the method above.

- **General Formula:**

$$\int \frac{dx}{x(x^n + a)} = \frac{1}{na} \log \left| \frac{x^n}{x^n + a} \right| + C$$

- **Application:** In this problem, $n = 5$ and $a = 3$.

$$I = \frac{1}{(5)(3)} \log \left| \frac{x^5}{x^5 + 3} \right| + C = \frac{1}{15} \log \left| \frac{x^5}{x^5 + 3} \right| + C$$

www.cuet.pw www.cbse.page

Conclusion

The calculated result matches option (b).

www.cuet.pw www.cbse.page

⋆ Answer

The final result is $\frac{1}{15} \log \left| \frac{x^5}{x^5 + 3} \right| + C$, which is option (b).

CUET 2022 PYQ 48

If $\int \frac{x^3}{x+1} dx = q(x) - \log|x+1| + C$ then $q(x)$ is equal to :

(a) $q(x) = \frac{x^3}{3} + x$

(b) $q(x) = \frac{x^2}{2} - x$

(c) $q(x) = x^2 - x + 1$

(d) $q(x) = \frac{x^3}{3} - \frac{x^2}{2} + x$

Concept Builder

www.cuet.pw www.cbse.page

★ **Concepts and Detailed Method**

www.cuet.pw www.cbse.page

1. Core Concept: Improper Rational Functions

The integrand is a rational function $\frac{P(x)}{Q(x)}$ where the degree of the numerator (3) is greater than the degree of the denominator (1).

- **Rule:** Whenever deg(numerator) $\geq$ deg(denominator), you must perform Polynomial Division (Long Division) or algebraic manipulation to simplify the expression before integrating.
- **Goal:** Convert $\frac{\text{Dividend}}{\text{Divisor}}$ into Quotient $+ \frac{\text{Remainder}}{\text{Divisor}}$.

www.cuet.pw www.cbse.page

2. Step-by-Step Solution

Step 1: Manipulate the Numerator We want to divide x^3 by $x+1$. A clever algebraic manipulation avoids long division. We know that x^3+1 is divisible by $x+1$. Add and subtract 1 in the numerator:

$$I = \int \frac{x^3}{x+1} dx = \int \frac{x^3+1-1}{x+1} dx$$

www.cuet.pw www.cbse.page

Step 2: Split the Fraction $I = \int \left(\frac{x^3+1}{x+1} - \frac{1}{x+1} \right) dx$

www.cuet.pw www.cbse.page

Step 3: Factorize Recall the algebraic identity for the sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$. Here, $x^3 + 1 = (x+1)(x^2 - x + 1)$. Substitute this back into the integral:

$$I = \int \left(\frac{(x+1)(x^2-x+1)}{x+1} - \frac{1}{x+1} \right) dx$$

www.cuet.pw www.cbse.page

Step 4: Cancel Terms

$$I = \int (x^2 - x + 1)\, dx - \int \frac{1}{x+1} dx$$

Step 5: Integrate Term by Term Using the power rule $\int x^n dx = \frac{x^{n+1}}{n+1}$:

$$\int x^2 dx = \frac{x^3}{3}$$

$$\int -x dx = -\frac{x^2}{2}$$

$$\int 1 dx = x$$

$$\int \frac{1}{x+1} dx = \log|x+1|$$

Combining these:

$$I = \left(\frac{x^3}{3} - \frac{x^2}{2} + x \right) - \log|x+1| + C$$

www.cuet.pw www.cbse.page

Step 6: Compare with the Question The question states: $I = q(x) - \log|x+1| + C$. Comparing our result with the given form:

$$\underbrace{\left(\frac{x^3}{3} - \frac{x^2}{2} + x \right)}_{q(x)} - \log|x+1| + C$$

Therefore:

$$q(x) = \frac{x^3}{3} - \frac{x^2}{2} + x$$

www.cuet.pw www.cbse.page

⋆ Short Tricks

www.cuet.pw www.cbse.page

Trick 1: Synthetic Division (Horner's Method)

If you struggle with the $+1/-1$ manipulation, use synthetic division to divide x^3 by $x+1$. Root is $x=-1$. Coefficients of x^3 are $1,0,0,0$.

-1	1	0	0	0
	$\downarrow$	-1	1	-1
	1	-1	1	**-1** (Remainder)

This gives the quotient $x^2 - x + 1$ and remainder -1. Integrate the quotient directly to get $q(x)$.

$$\int (x^2 - x + 1)\,dx = \frac{x^3}{3} - \frac{x^2}{2} + x$$

www.cuet.pw www.cbse.page

Trick 2: Derivative Check (Reverse Engineering)

Since integration is the reverse of differentiation, differentiation is the reverse of integration.

$$\frac{d}{dx}[\text{RHS}] = \text{Integrand}$$

$$\frac{d}{dx}[q(x) - \log(x+1)] = \frac{x^3}{x+1}$$

$$q'(x) - \frac{1}{x+1} = \frac{x^3}{x+1}$$

$$q'(x) = \frac{x^3}{x+1} + \frac{1}{x+1} = \frac{x^3+1}{x+1} = x^2 - x + 1$$

Now, simply integrate $x^2 - x + 1$ to find $q(x)$. This avoids the initial confusion of how to split the fraction.

www.cuet.pw www.cbse.page

⋆ Final Answer

The function $q(x)$ is $\frac{x^3}{3} - \frac{x^2}{2} + x$. This corresponds to option **(d)**.

CUET 2022 PYQ 49

$$\int_{-1}^{1} (|x-2| + |x|)\,dx =$$

www.cuet.pw www.cbse.page

(a) 7 (b) 5 (c) 4 (d) 6

Concept Builder

www.cuet.pw www.cbse.page

⋆ Detailed Concept and Method

www.cuet.pw www.cbse.page

1. Understanding the Modulus Function

The definition of the modulus function $|x|$ is crucial here:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

For a shifted function like $|x-a|$:

$$|x-a| = \begin{cases} x-a & \text{if } x \geq a \\ -(x-a) & \text{if } x < a \end{cases}$$

www.cuet.pw www.cbse.page

2. Analyzing the Critical Points

We need to remove the modulus signs to integrate. To do this, we find the points where the terms inside the modulus become zero (critical points).

- For $|x|$, the critical point is $x = 0$.
- For $|x-2|$, the critical point is $x = 2$.

www.cuet.pw www.cbse.page

3. Step-by-Step Solution

Step 1: Analyze the Integrand within the Limits $[-1,1]$ The integration limits are from $x=-1$ to $x=1$.

(1) Analyze $|x-2|$:

Since our interval is $[-1,1]$, the value of x is always less than 2 (i.e., $x<2$).

Therefore, $|x-2|$ will always open with a negative sign in this interval.

$$|x-2| = -(x-2) = 2-x \quad \text{for } x \in [-1,1]$$

www.cuet.pw www.cbse.page

(2) Analyze $|x|$:

The critical point $x=0$ lies **inside** the interval $[-1,1]$. We must split the integral at $x=0$.

$$|x| = \begin{cases} -x & \text{for } -1 \le x < 0 \\ x & \text{for } 0 \le x \le 1 \end{cases}$$

www.cuet.pw www.cbse.page

Step 2: Split the Integral We break the integral $I = \int_{-1}^{1} (|x-2| + |x|)\,dx$ at $x=0$:

$$I = \int_{-1}^{0} (\underbrace{2-x}_{|x-2|} + \underbrace{(-x)}_{|x|})\,dx + \int_{0}^{1} (\underbrace{2-x}_{|x-2|} + \underbrace{x}_{|x|})\,dx$$

www.cuet.pw www.cbse.page

Step 3: Simplify the Integrands

$$I = \int_{-1}^{0} (2-2x)\,dx + \int_{0}^{1} (2)\,dx$$

www.cuet.pw www.cbse.page

Step 4: Evaluate the Integrals

$$I = \left[2x - \frac{2x^2}{2}\right]_{-1}^{0} + [2x]_{0}^{1}$$

$$I = \left[2x - x^2\right]_{-1}^{0} + [2(1) - 2(0)]$$

www.cuet.pw www.cbse.page

Upper and Lower Limits: For the first part: Upper Limit (0): $2(0) - 0^2 = 0$
Lower Limit (-1): $2(-1) - (-1)^2 = -2 - 1 = -3$
Value: $0 - (-3) = 3$
For the second part: Value: 2

$$I = 3 + 2 = 5$$

www.cuet.pw www.cbse.page

⋆ Short Trick: Geometry Method

Definite integration represents the **Area under the curve**. Since the functions inside the modulus are linear, their graphs form straight lines, triangles, or trapezoids.

www.cuet.pw www.cbse.page

Let's calculate the area geometrically without integration formulas.

- **Area 1:** $\int_{-1}^{1} |x|dx$ The graph of $|x|$ from -1 to 1 forms two identical right-angled triangles.
  - Triangle 1 (left): Base = 1, Height = $|-1| = 1$. Area = $\frac{1}{2} \times 1 \times 1 = 0.5$
  - Triangle 2 (right): Base = 1, Height = $|1| = 1$. Area = $\frac{1}{2} \times 1 \times 1 = 0.5$
  - Total Area for $|x| = 0.5 + 0.5 = 1$.

www.cuet.pw www.cbse.page

- **Area 2:** $\int_{-1}^{1} |x-2|dx$ Since $x < 2$ in this interval, this is the line $y = 2 - x$. This forms a Trapezoid between $x = -1$ and $x = 1$.
  - Height (at $x = -1$) = $|-1-2| = 3$
  - Height (at $x = 1$) = $|1-2| = 1$
  - Width of interval = $1 - (-1) = 2$
  - Area of Trapezoid = $\frac{1}{2}$(sum of parallel sides) × width
  - Area = $\frac{1}{2}(3+1) \times 2 = 4$

www.cuet.pw www.cbse.page

- **Total Area:**

$$\text{Total} = \text{Area}(|x|) + \text{Area}(|x-2|) = 1 + 4 = 5$$

⋆ Final Answer

The value of the integral is 5. Correct Option: **(b)**

CUET 2022 PYQ 50

$$\int_{-1}^{1} e^{|x|}dx =$$

www.cuet.pw www.cbse.page

(a) $2(e^{-1}-1)$ (b) $2(e+1)$ (c) $e-1$ (d) $2(e-1)$

Concept Builder

www.cuet.pw www.cbse.page

Goal: To evaluate the definite integral $\int_{-1}^{1} e^{|x|}dx$ and identify the correct option.

www.cuet.pw www.cbse.page

Core Concept: The Modulus Function

www.cuet.pw www.cbse.page

The modulus function $|x|$ changes its definition based on the sign of x:

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Because the integrand involves $|x|$, and the interval $[-1, 1]$ crosses zero, we must handle the negative and positive parts separately.
[Image of graph of exponential modulus function]

www.cuet.pw www.cbse.page

Method 1: Splitting the Integral (Standard Method)

www.cuet.pw www.cbse.page

We split the integral at the point where the definition of $|x|$ changes, which is $x = 0$.

$$I = \int_{-1}^{1} e^{|x|}dx = \int_{-1}^{0} e^{|x|}dx + \int_{0}^{1} e^{|x|}dx$$

- **Step 1: Substitute the definition of $|x|$**

  - In the interval $[-1, 0]$, x is negative, so $|x| = -x$.

- In the interval $[0,1]$, x is positive, so $|x| = x$.

$$I = \int_{-1}^{0} e^{-x}dx + \int_{0}^{1} e^{x}dx$$

- **Step 2: Integrate each part**

- First Integral: $\int e^{-x}dx = -e^{-x}$.
- Second Integral: $\int e^{x}dx = e^{x}$.

$$I = \left[-e^{-x}\right]_{-1}^{0} + \left[e^{x}\right]_{0}^{1}$$

- **Step 3: Apply Limits**

- For the first part: $(-e^{-0}) - (-e^{-(-1)}) = (-1) - (-e^{1}) = e - 1$
- For the second part: $(e^{1}) - (e^{0}) = e - 1$

- **Step 4: Sum the results**

$$I = (e-1) + (e-1)$$

$$I = 2(e-1)$$

www.cuet.pw www.cbse.page

Method 2: Even Function Property (Short Trick)

www.cuet.pw www.cbse.page

This method uses symmetry to simplify the calculation and is faster.

- **Concept:** A function $f(x)$ is an **Even Function** if $f(-x) = f(x)$. The graph of an even function is symmetric about the y-axis. Property: $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.

- **Step 1: Check for Evenness** Let $f(x) = e^{|x|}$.

$$f(-x) = e^{|-x|} = e^{|x|} = f(x)$$

Since $f(-x) = f(x)$, the function is even.

- **Step 2: Apply the Property** Instead of integrating from -1 to 1, we double the integral from 0 to 1.

$$\int_{-1}^{1} e^{|x|}dx = 2\int_{0}^{1} e^{|x|}dx$$

- **Step 3: Solve simple integral** In the range $[0, 1]$, $|x| = x$.

$$I = 2\int_0^1 e^x dx$$

$$I = 2\left[e^x\right]_0^1$$

$$I = 2(e^1 - e^0)$$

$$I = 2(e - 1)$$

www.cuet.pw www.cbse.page

Conclusion

www.cuet.pw www.cbse.page

The value of the integral is $2(e - 1)$.
The correct option is **(d)**.

CUET 2022 PYQ 51

Match List I with List II

www.cuet.pw www.cbse.page

List I

(A) $\int \frac{1}{x+\sqrt{x}} dx$

(B) $\int \frac{e^{\log \sqrt{x}}}{x} dx$

(C) $\int \frac{dx}{4x^2-9}$

(D) $\int e^{\sqrt{x}} dx$

List II

I. $2\sqrt{x}+C$

II. $2(\sqrt{x}-1)e^{\sqrt{x}}+C$

III. $2\log\left(\sqrt{x}+1\right)+C$

IV. $\frac{1}{12}\log\left|\frac{2x-3}{2x+3}\right|+C$

Choose the correct answer from the options given below:

www.cuet.pw www.cbse.page

(a) A - II, B - IV, C - I, D - III

(b) A - III, B - II, C - IV, D - I

(c) A - III, B - I, C - IV, D - II

(d) A - I, B - II, C - III, D - IV

Concept Builder

www.cuet.pw www.cbse.page

Goal: To evaluate four distinct indefinite integrals and match them with their corresponding solutions from List II.

www.cuet.pw www.cbse.page

Detailed Step-by-Step Solutions

www.cuet.pw www.cbse.page

Integral (A): $\int \frac{1}{x+\sqrt{x}}dx$

- **Concept:** Integration by Substitution. When terms like x and $\sqrt{x}$ appear together, factoring out the lower power often helps.
- **Step 1: Factor the Denominator**

$$I = \int \frac{1}{\sqrt{x}(\sqrt{x}+1)}dx$$

- **Step 2: Substitute** Let $t = \sqrt{x}+1$. Differentiating:

$$dt = \frac{1}{2\sqrt{x}}dx \implies 2dt = \frac{1}{\sqrt{x}}dx.$$

- **Step 3: Integrate** Substitute back into the integral:

$$I = \int \frac{2dt}{t} = 2\log|t| + C$$

- **Step 4: Replace variable**

$$I = 2\log\left(\sqrt{x}+1\right) + C$$

- **Match: A → III**

www.cuet.pw www.cbse.page

Integral (B): $\int \frac{e^{\log\sqrt{x}}}{x}dx$

- **Concept:** Logarithmic Identity. Recall that $e^{\log f(x)} = f(x)$.
- **Step 1: Simplify the Numerator**

$$e^{\log\sqrt{x}} = \sqrt{x}$$

The integral becomes:

$$I = \int \frac{\sqrt{x}}{x}dx = \int x^{1/2-1}dx = \int x^{-\frac{1}{2}}dx$$

- **Step 2: Integrate using Power Rule**

$$I = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{x^{1/2}}{\frac{1}{2}} + C$$

$$I = 2\sqrt{x} + C$$

- **Match: B → I**

www.cuet.pw www.cbse.page

Integral (C): $\int \frac{dx}{4x^2 - 9}$

- **Concept:** Standard Integral Formula.

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

- **Step 1: Rewrite in Standard Form** Make the coefficient of x^2 equal to 1 by factoring out 4, OR treat it as $(2x)^2$. Let's use $(2x)^2$.

$$I = \int \frac{dx}{(2x)^2 - (3)^2}$$

- **Step 2: Apply Formula** Here, variable is $2x$ (so we must divide by the coefficient 2) and $a = 3$.

$$I = \frac{1}{2(\text{coeff of } x)} \cdot \frac{1}{2a} \log \left| \frac{2x-3}{2x+3} \right| + C$$

$$I = \frac{1}{2} \cdot \frac{1}{2(3)} \log \left| \frac{2x-3}{2x+3} \right| + C$$

$$I = \frac{1}{12} \log \left| \frac{2x-3}{2x+3} \right| + C$$

- **Match: C → IV**

www.cuet.pw www.cbse.page

Integral (D): $\int e^{\sqrt{x}} dx$

- **Concept:** Substitution followed by Integration by Parts.

- **Step 1: Substitution** Let $t = \sqrt{x} \implies x = t^2 \implies dx = 2t\,dt$.

$$I = \int e^t \cdot 2t\,dt = 2\int te^t\,dt$$

- **Step 2: Integration by Parts** Using ILATE rule, $u = t$ (Algebraic) and $dv = e^t\,dt$ (Exponential). Formula: $\int u\,dv = uv - \int v\,du$.

$$\int te^t\,dt = te^t - \int e^t\,dt = te^t - e^t = e^t(t-1)$$

- **Step 3: Final Answer** Multiply by the constant 2 and substitute $t = \sqrt{x}$.

$$I = 2e^{\sqrt{x}}(\sqrt{x}-1) + C$$

- **Match: D → II**

www.cuet.pw www.cbse.page

Final Matching Configuration

www.cuet.pw www.cbse.page

- A → III
- B → I
- C → IV
- D → II

Checking the options:

- (a) A - II... (Incorrect)
- (b) A - III, B - II... (Incorrect B match)
- (c) A - III, B - I, C - IV, D - II (Correct)
- (d) A - I... (Incorrect)

www.cuet.pw www.cbse.page

Conclusion

www.cuet.pw www.cbse.page

The correct option is **(c)**.

CUET 2022 PYQ 52

$$\frac{d}{dx}\left[\int_0^{2a} f(\sin 2x)\,dx\right] =$$

www.cuet.pw www.cbse.page

(a) $2a$ (b) $f(\sin 2a)$ (c) $f(\cos 2a)$ (d) 0

Concept Builder

www.cuet.pw www.cbse.page

Goal: To find the value of the derivative of the definite integral:

$$\frac{d}{dx}\left[\int_0^{2a} f(\sin 2x)\,dx\right]$$

www.cuet.pw www.cbse.page

Core Concept: Derivative of a Definite Integral

www.cuet.pw www.cbse.page

To solve this, we must understand what a definite integral represents.

- **Definite Integral as a Constant:** A definite integral $\int_A^B g(t)\,dt$ represents the area under the curve $g(t)$ from $t = A$ to $t = B$. If the limits A and B are constants (numbers that do not change), then the result of the integral is a constant numerical value.

- **Derivative of a Constant:** The derivative of any constant value with respect to a variable is always zero.

$$\frac{d}{dx}(\text{Constant}) = 0$$

www.cuet.pw www.cbse.page

Detailed Step-by-Step Solution

www.cuet.pw www.cbse.page

- **Step 1: Analyze the Integral Term** Let I be the integral inside the bracket.

$$I = \int_0^{2a} f(\sin 2x)\,dx$$

Notice the limits of integration:

- Lower limit = 0 (Constant)
- Upper limit = $2a$ (Constant, since 'a' is an arbitrary constant)

Because the limits are fixed constants, solving this integral will yield a specific numerical value (let's call it K) that depends on a, but **does not depend on the variable** x. The x inside the integral is just a "dummy variable" that gets integrated out.

$$\text{So, } \int_0^{2a} f(\sin 2x)\,dx = \text{Constant Value } (K)$$

- **Step 2: Differentiate** The question asks for the derivative of this result with respect to x.

$$\frac{d}{dx}[I] = \frac{d}{dx}(\text{Constant})$$

Since the derivative of a constant is zero:

$$\frac{d}{dx}\left[\int_0^{2a} f(\sin 2x)\,dx\right] = 0$$

www.cuet.pw www.cbse.page

Short Trick (Visual Check)

www.cuet.pw www.cbse.page

- Look at the limits of the integral: **0** and **2a**.
- Are there any variables (like x) in the limits? **No**.
- If the limits are constants, the integral is a constant.
- The derivative of a constant is always **0**.
- **Answer is 0.**

www.cuet.pw www.cbse.page

Conclusion

www.cuet.pw www.cbse.page

The value is 0.

The correct option is **(d)**.

CUET 2022 PYQ 53

$$\int \tan x(\sec x - \tan x)\,dx =$$

www.cuet.pw www.cbse.page

(a) $\sec x - \tan x + x - C$

(b) $\sec x - \tan^2 x + C$

(c) $\sec x + \tan x + x + C$

(d) $\sec x - \tan x + C$

Concept Builder

www.cuet.pw www.cbse.page

Goal: To solve the indefinite integral $\int \tan x(\sec x - \tan x)\,dx$.

www.cuet.pw www.cbse.page

Core Concepts: Standard Integrals & Trigonometric Identities

www.cuet.pw www.cbse.page

To solve this, we need three key pieces of information:

- **Distributive Property:** $a(b-c) = ab - ac$.
- **Standard Integrals:**
  - $\int \sec x \tan x\,dx = \sec x$ (since $\frac{d}{dx}(\sec x) = \sec x \tan x$).
  - $\int \sec^2 x\,dx = \tan x$ (since $\frac{d}{dx}(\tan x) = \sec^2 x$).
  - $\int 1\,dx = x$.
- **Trigonometric Identity:** $\sec^2 x - \tan^2 x = 1 \implies \tan^2 x = \sec^2 x - 1$.

www.cuet.pw www.cbse.page

Detailed Step-by-Step Solution

www.cuet.pw www.cbse.page

- **Step 1: Expand the Integrand** Multiply $\tan x$ with the terms inside the bracket:

$$I = \int (\tan x \cdot \sec x - \tan x \cdot \tan x)\, dx$$

$$I = \int (\sec x \tan x - \tan^2 x)\, dx$$

- **Step 2: Convert** $\tan^2 x$ Since we don't have a direct standard integral for $\tan^2 x$, we use the identity $\tan^2 x = \sec^2 x - 1$. Substitute this into the integral:

$$I = \int [\sec x \tan x - (\sec^2 x - 1)]\, dx$$

$$I = \int (\sec x \tan x - \sec^2 x + 1)\, dx$$

- **Step 3: Integrate Term by Term** Split the integral into three parts:

$$I = \int \sec x \tan x \, dx - \int \sec^2 x \, dx + \int 1 \, dx$$

- **Step 4: Apply Standard Formulas**

$$I = (\sec x) - (\tan x) + (x) + C$$

$$I = \sec x - \tan x + x + C$$

- **Step 5: Match with Options** Let's look at the options provided:

(a) $\sec x - \tan x + x - C$

(b) $\sec x - \tan^2 x + C$

(c) $\sec x + \tan x + x + C$

(d) $\sec x - \tan x + C$

Our result is $\sec x - \tan x + x + C$.

Note on the Constant: In indefinite integration, the constant of integration represents an arbitrary value. Therefore, $+C$, $-C$, or $+K$ essentially mean the same thing (an arbitrary constant). Option (a) matches our result perfectly in terms of variable terms $(\sec x - \tan x + x)$.

www.cuet.pw www.cbse.page

Short Trick (Mental Flow)

www.cuet.pw www.cbse.page

- **Expand:** You see $\sec x \tan x$ (Derivative of $\sec x$) and $-\tan^2 x$.
- **Transform:** Think of $-\tan^2 x$ as $-(\sec^2 x - 1) = -\sec^2 x + 1$.
- **Integrate:**
  - ✓ $\sec x \tan x \to \sec x$
  - ✓ $-\sec^2 x \to -\tan x$
  - ✓ $+1 \to +x$
- **Combine:** $\sec x - \tan x + x$.

www.cuet.pw www.cbse.page

Conclusion

www.cuet.pw www.cbse.page

The correct answer is $\sec x - \tan x + x - C$.

The correct option is **(a)**.

CUET 2022 PYQ 54

$$\int_1^2 \frac{dx}{x(x^4+1)} = ?$$

www.cuet.pw www.cbse.page

(a) $\log\left(\frac{32}{17}\right)$ (b) $\log\left(\frac{16}{17}\right)$ (c) $\frac{1}{4}\log\left(\frac{16}{17}\right)$ (d) $\frac{1}{4}\log\left(\frac{32}{17}\right)$

Concept Builder

www.cuet.pw www.cbse.page

Goal: To evaluate the definite integral $\int_1^2 \frac{dx}{x(x^4+1)}$.

www.cuet.pw www.cbse.page

Method 1: Substitution (Standard Approach)

www.cuet.pw www.cbse.page

The integrand involves terms like x and x^4. A standard technique for integrals of the form $\frac{1}{x(x^n+1)}$ is to multiply the numerator and denominator by x^{n-1} to create a substitution opportunity.

- **Step 1: Manipulate the Integrand** Multiply numerator and denominator by x^3:

$$I = \int_1^2 \frac{x^3}{x^3 \cdot x(x^4+1)} dx$$

$$I = \int_1^2 \frac{x^3}{x^4(x^4+1)} dx$$

- **Step 2: Substitution** Let $t = x^4$. Differentiate with respect to x:

$$dt = 4x^3 dx \implies \frac{dt}{4} = x^3 dx$$

- **Step 3: Change the Limits**

  - When $x = 1 \implies t = 1^4 = 1$.
  - When $x = 2 \implies t = 2^4 = 16$.

- **Step 4: Rewrite the Integral in terms of t**

$$I = \int_1^{16} \frac{1}{t(t+1)} \cdot \frac{dt}{4}$$

$$I = \frac{1}{4} \int_1^{16} \frac{1}{t(t+1)} dt$$

- **Step 5: Partial Fractions (Short Method)**

Notice that $\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$.

$$I = \frac{1}{4} \int_1^{16} \left(\frac{1}{t} - \frac{1}{t+1} \right) dt$$

- **Step 6: Integrate**

$$I = \frac{1}{4} \left[\log|t| - \log|t+1| \right]_1^{16}$$

Using log property $\log a - \log b = \log\left(\frac{a}{b}\right)$:

$$I = \frac{1}{4} \left[\log\left(\frac{t}{t+1}\right) \right]_1^{16}$$

- **Step 7: Apply Limits**

$$I = \frac{1}{4} \left[\log\left(\frac{16}{16+1}\right) - \log\left(\frac{1}{1+1}\right) \right]$$

$$I = \frac{1}{4} \left[\log\left(\frac{16}{17}\right) - \log\left(\frac{1}{2}\right) \right]$$

Using log property $\log\left(\frac{a}{b}\right) - \log\left(\frac{c}{d}\right) = \log\left(\frac{a}{b} \nabla \cdot \frac{c}{d}\right) = \log\left(\frac{ad}{bc}\right)$:

$$I = \frac{1}{4} \log\left(\frac{16}{17} \times \frac{2}{1}\right)$$

$$I = \frac{1}{4} \log\left(\frac{32}{17}\right)$$

www.cuet.pw www.cbse.page

Method 2: Short Trick (Common Form)

www.cuet.pw www.cbse.page

There is a standard formula for this specific type of integral:

- **Formula:**

$$\int \frac{dx}{x(x^n+1)} = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$$

- **Apply Directly:** Here, $n = 4$.

$$I = \left[\frac{1}{4}\log\left(\frac{x^4}{x^4+1}\right)\right]_1^2$$

- **Plug in Limits:** Upper Limit ($x = 2$): $\frac{1}{4}\log\left(\frac{16}{17}\right)$ Lower Limit ($x = 1$): $\frac{1}{4}\log\left(\frac{1}{2}\right)$
- **Subtract:**

$$\frac{1}{4}\left[\log\left(\frac{16}{17}\right) - \log\left(\frac{1}{2}\right)\right]$$

$$\frac{1}{4}\log\left(\frac{16/17}{1/2}\right) = \frac{1}{4}\log\left(\frac{32}{17}\right)$$

www.cuet.pw www.cbse.page

Conclusion

www.cuet.pw www.cbse.page

The value of the integral is $\frac{1}{4}\log\left(\frac{32}{17}\right)$.

The correct option is **(d)**.

CUET 2022 PYQ 55

$$\int \frac{2}{x^4-1}\,dx =$$

www.cuet.pw www.cbse.page

(A) $\log\left|\frac{x^2-1}{x^2+1}\right| + C$

(B) $2\tan^{-1}\left(x^2\right) + C$

(C) $\frac{1}{2}\log\left|\frac{x-1}{x+1}\right| - \tan^{-1}x + C$

(D) $\tan^{-1}x + \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| + C$

Concept Builder

www.cuet.pw www.cbse.page

Teaching Guide and Concepts

To solve this integration problem efficiently, we need to decompose the complex rational function into simpler fractions that can be integrated using standard formulas.

- **Factorization of Denominator:** The term $x^4 - 1$ is a difference of squares. It can be factored as:

$$x^4 - 1 = \left(x^2\right)^2 - 1^2 = \left(x^2-1\right)\left(x^2+1\right)$$

- **Partial Fraction Decomposition (The Split Trick):** Instead of finding variables A, B, C, D immediately, look for a relationship between the factors in the denominator. Notice that the difference between the factors $\left(x^2+1\right)$ and $\left(x^2-1\right)$ is exactly equal to the numerator constant:

$$\left(x^2+1\right) - \left(x^2-1\right) = 2$$

This observation allows us to split the integral instantly without long calculations.

- **Standard Integration Formulas:** We will need two fundamental integration formulas:

(i) $$\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$

(ii) $$\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

www.cuet.pw www.cbse.page

Step-by-Step Solution

Let the integral be I.

$$I = \int \frac{2}{x^4 - 1}\,dx$$

- **Step 1: Factorize the denominator**

$$I = \int \frac{2}{(x^2-1)(x^2+1)}\,dx$$

- **Step 2: Apply the "Split Trick"** Rewrite the numerator 2 as $[(x^2+1)-(x^2-1)]$.

$$I = \int \frac{(x^2+1)-(x^2-1)}{(x^2-1)(x^2+1)}\,dx$$

- **Step 3: Separate the terms** Divide each term in the numerator by the denominator:

$$I = \int \left(\frac{x^2+1}{(x^2-1)(x^2+1)} - \frac{x^2-1}{(x^2-1)(x^2+1)}\right)dx$$

Cancel the common terms:

$$I = \int \left(\frac{1}{x^2-1} - \frac{1}{x^2+1}\right)dx$$

- **Step 4: Integrate each part separately**

$$I = \int \frac{1}{x^2-1}\,dx - \int \frac{1}{x^2+1}\,dx$$

Apply the standard formulas identified in the teaching guide (with $a = 1$):

(1) For the first part: $\int \frac{1}{x^2-1^2}\,dx = \frac{1}{2(1)}\log\left|\frac{x-1}{x+1}\right|$

(2) For the second part: $\int \frac{1}{x^2+1^2}\,dx = \tan^{-1}x$

- **Step 5: Combine the results**

$$I = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| - \tan^{-1}x + C$$

Therefore, the correct option is **(C)**.

www.cuet.pw www.cbse.page

Alternative Method (Standard Partial Fractions)

If the numerator trick doesn't strike you, you can use the formal method.

- **Setup:**

$$\frac{2}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

- **Find Coefficients:**

(i) To find A, hide $(x-1)$ and put $x=1$: $A = \frac{2}{(1+1)(1+1)} = \frac{2}{4} = \frac{1}{2}$.

(ii) To find B, hide $(x+1)$ and put $x=-1$:
$B = \frac{2}{(-1-1)(1+1)} = \frac{2}{-4} = -\frac{1}{2}$.

(iii) By comparing coefficients, you would find $C=0$ and $D=-1$.

- **Integrate:**

$$\int \left(\frac{1/2}{x-1} - \frac{1/2}{x+1} - \frac{1}{x^2+1}\right)dx$$

$$= \frac{1}{2}\left[\log|x-1| - \log|x+1|\right] - \tan^{-1}x$$

$$= \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| - \tan^{-1}x$$

This yields the same result but takes slightly longer.

CUET 2022 PYQ 56

$$\int_{1/4}^{1} \frac{dx}{\sqrt{-x^2-2x+3}} =$$

www.cuet.pw www.cbse.page

(a) $\sin^{-1}\left(\frac{1}{4}\right)$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$ (c) $\sin^{-1}\left(\frac{5}{8}\right)$ (d) $\cos^{-1}\left(\frac{5}{8}\right)$

Concept Builder

www.cuet.pw www.cbse.page

Teaching Guide

- **Completing the Square:** When the denominator contains a quadratic expression under a square root (like $\sqrt{ax^2+bx+c}$), the standard method is to "complete the square" to transform it into the form $\sqrt{a^2-x^2}$ or $\sqrt{x^2 \pm a^2}$. For an expression like $-x^2-2x+3$, factor out the negative sign from the x terms first: $-\left(x^2+2x\right)+3$. Then add and subtract the square of half the coefficient of x.
- **Standard Integration Formula:** The specific form we aim for here is:

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

- **Inverse Trigonometric Identity:** Sometimes the calculated answer might be in terms of $\sin^{-1}$, but the options are in $\cos^{-1}$. Remember the complementary angle identity:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

Therefore, $\frac{\pi}{2} - \sin^{-1} x = \cos^{-1} x$.

www.cuet.pw www.cbse.page

Detailed Solution

- **Step 1: Simplify the Quadratic Expression** We need to rewrite the term inside the square root: $-x^2-2x+3$. Take the negative sign

common from the x terms:

$$-x^2-2x+3=-\left(x^2+2x\right)+3$$

Add and subtract 1 inside the bracket to complete the square:

$$-\left(x^2+2x+1-1\right)+3$$

$$-\left((x+1)^2-1\right)+3$$

$$-(x+1)^2+1+3$$

$$4-(x+1)^2$$

$$2^2-(x+1)^2$$

- **Step 2: Rewrite the Integral** Substitute this back into the integral:

$$I=\int_{1/4}^{1}\frac{dx}{\sqrt{2^2-(x+1)^2}}$$

- **Step 3: Integrate** Using the formula $\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)$: Here, our variable is $(x+1)$ and $a=2$.

$$I=\left[\sin^{-1}\left(\frac{x+1}{2}\right)\right]_{\frac{1}{4}}^{1}$$

- **Step 4: Apply Limits** Upper Limit ($x=1$):

$$\sin^{-1}\left(\frac{1+1}{2}\right)=\sin^{-1}\left(\frac{2}{2}\right)=\sin^{-1}(1)=\frac{\pi}{2}$$

Lower Limit ($x=\frac{1}{4}$):

$$\sin^{-1}\left(\frac{\frac{1}{4}+1}{2}\right)=\sin^{-1}\left(\frac{\frac{5}{4}}{2}\right)=\sin^{-1}\left(\frac{5}{8}\right)$$

Value of Integral I:

$$I=\text{Upper Limit}-\text{Lower Limit}$$

$$I=\frac{\pi}{2}-\sin^{-1}\left(\frac{5}{8}\right)$$

- **Step 5: Match with Options (Using Identity)** Looking at the options, we don't see $\frac{\pi}{2} - \sin^{-1}\left(\frac{5}{8}\right)$. However, we know that $\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$. Therefore:

$$\frac{\pi}{2} - \sin^{-1}\left(\frac{5}{8}\right) = \cos^{-1}\left(\frac{5}{8}\right)$$

Thus, the correct option is **(D)**.

www.cuet.pw www.cbse.page

Alternative Methods & Short Tricks

- **Substitution Method:** You can substitute $x + 1 = 2\sin\theta$. Then $dx = 2\cos\theta\, d\theta$. Limits change: When $x = 1 \Rightarrow 2\sin\theta = 2 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$. When $x = 1/4 \Rightarrow 2\sin\theta = 5/4 \Rightarrow \sin\theta = 5/8 \Rightarrow \theta = \sin^{-1}(5/8)$. This leads directly to $\int d\theta = [\theta]$, which gives the same result.

CUET 2022 PYQ 57

If $\int \frac{dx}{\sqrt{x+2}-\sqrt{x+1}} = \frac{2}{3}\left[(\lambda+1)^{3/2}-\lambda^{3/2}\right]+C$, then the value of λ is

www.cuet.pw www.cbse.page

(a) $x-1$ (b) x (c) $x+1$ (d) $\frac{1}{x}$

Concept Builder

www.cuet.pw www.cbse.page

Solution:

- **1. Rationalize the Denominator**

  The integrand is in the form $\frac{1}{\sqrt{A}-\sqrt{B}}$. To simplify this, we must rationalize the denominator by multiplying the numerator and denominator by the conjugate term $\left(\sqrt{x+2}+\sqrt{x+1}\right)$.

$$I = \int \frac{dx}{\sqrt{x+2}-\sqrt{x+1}}$$
$$= \int \frac{1}{\sqrt{x+2}-\sqrt{x+1}} \times \frac{\sqrt{x+2}+\sqrt{x+1}}{\sqrt{x+2}+\sqrt{x+1}}\,dx$$

- **2. Simplify the Expression**

  Using the algebraic identity $(a-b)(a+b) = a^2-b^2$ in the denominator:

$$I = \int \frac{\sqrt{x+2}+\sqrt{x+1}}{\left(\sqrt{x+2}\right)^2-\left(\sqrt{x+1}\right)^2}\,dx$$
$$= \int \frac{\sqrt{x+2}+\sqrt{x+1}}{(x+2)-(x+1)}\,dx$$
$$= \int \frac{\sqrt{x+2}+\sqrt{x+1}}{1}\,dx$$
$$= \int \left[(x+2)^{1/2}+(x+1)^{1/2}\right]\,dx$$

- **3. Perform Integration**

We use the standard power rule for integration:

$$\int (x+a)^n \, dx = \frac{(x+a)^{n+1}}{n+1}.$$

$$I = \frac{(x+2)^{1/2+1}}{\frac{1}{2}+1} + \frac{(x+1)^{1/2+1}}{\frac{1}{2}+1} + C$$

$$= \frac{(x+2)^{3/2}}{\frac{3}{2}} + \frac{(x+1)^{3/2}}{\frac{3}{2}} + C$$

$$= \frac{2}{3}(x+2)^{3/2} + \frac{2}{3}(x+1)^{3/2} + C$$

- **4. Compare with Given Equation**

  The problem states the result is:

$$\frac{2}{3}\left[(\lambda+1)^{3/2} - \lambda^{3/2}\right] + C$$

  Our calculated result is:

$$\frac{2}{3}\left[(x+2)^{3/2} + (x+1)^{3/2}\right] + C$$

Note: There appears to be a sign typo in the question's denominator or the resulting expression (minus vs plus). However, by comparing the base terms structurally: The term $(\lambda+1)$ corresponds to $(x+2)$. The term λ corresponds to $(x+1)$.

$$\therefore \lambda = x+1$$

www.cuet.pw **www.cbse.page**

- **Correct Option**

  The value of λ is $x+1$. Hence, the correct option is **(C)**.

Teaching Guide & Key Concepts:

- **Rationalization of Surds**

  Whenever you encounter an integral of the form $\int \frac{1}{\sqrt{f(x)} \pm \sqrt{g(x)}} \, dx$,

  the primary method is rationalization. **Technique:** Multiply and divide by the conjugate.

(i) If denominator is $\sqrt{A}-\sqrt{B}$, multiply by $\dfrac{\sqrt{A}+\sqrt{B}}{\sqrt{A}+\sqrt{B}}$.

(ii) If denominator is $\sqrt{A}+\sqrt{B}$, multiply by $\dfrac{\sqrt{A}-\sqrt{B}}{\sqrt{A}-\sqrt{B}}$.

This process usually removes the square roots from the denominator, turning it into a constant or a simple polynomial.

- **Integration Power Rule**

  Recall that for linear arguments:

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$$

  In this problem, $a = 1$, making the application direct.

- **Short Trick (Pattern Recognition)**

  For integrals $\displaystyle\int \frac{dx}{\sqrt{x+a}-\sqrt{x+b}}$, if the difference inside the roots is constant (i.e., $(x+a)-(x+b)=k$), the answer follows the pattern:

$$\frac{2}{3k}\left[(x+a)^{3/2}+(x+b)^{3/2}\right]+C$$

  Here, $k=(x+2)-(x+1)=1$. So the answer is directly $\dfrac{2}{3}\left[(x+2)^{3/2}+(x+1)^{3/2}\right]$. Comparing this with the given λ form immediately reveals $\lambda = x+1$.

CUET 2022 PYQ 58

$$\int_0^{\pi} \sin^3 x \cdot \cos^2 x \, dx =$$

www.cuet.pw www.cbse.page

(a) $-\frac{4}{15}$ (b) $\frac{2}{15}$ (c) 0 (d) $\frac{4}{15}$

Concept Builder

www.cuet.pw www.cbse.page

Solution:

- **1. Analyze the Integrand and Method**

  Let the given integral be I.

$$I = \int_0^{\pi} \sin^3 x \cdot \cos^2 x \, dx$$

  Since the power of $\sin x$ is odd (3), it is efficient to keep one $\sin x$ factor for the differential dx and convert the remaining $\sin^2 x$ into $\cos x$.

$$I = \int_0^{\pi} \sin^2 x \cdot \cos^2 x \cdot \sin x \, dx$$

  Using the identity $\sin^2 x = 1 - \cos^2 x$:

$$I = \int_0^{\pi} (1 - \cos^2 x) \cos^2 x \cdot \sin x \, dx$$

- **2. Apply Substitution Method**

  Let $\cos x = t$.

  Differentiating with respect to x:

$$-\sin x \, dx = dt \implies \sin x \, dx = -dt$$

  Now, we change the limits of integration:

  (i) When $x = 0$, $t = \cos 0 = 1$.

  (ii) When $x = \pi$, $t = \cos \pi = -1$.

- **3. Substitute and Simplify**

  Substituting the values into the integral:

$$I = \int_{1}^{-1} \left(1 - t^2\right) t^2 (-dt)$$

  Using the property $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ to flip limits and absorb the negative sign:

$$I = \int_{-1}^{1} \left(t^2 - t^4\right) dt$$

- **4. Evaluate the Definite Integral**

  Since $f(t) = t^2 - t^4$ is an even function (because $f(-t) = (-t)^2 - (-t)^4 = t^2 - t^4 = f(t)$), we can use the property $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

$$\begin{aligned}
I &= 2\int_0^1 \left(t^2 - t^4\right) dt \\
&= 2\left[\frac{t^3}{3} - \frac{t^5}{5}\right]_0^1 \\
&= 2\left[\left(\frac{1}{3} - \frac{1}{5}\right) - (0 - 0)\right] \\
&= 2\left[\frac{5-3}{15}\right] \\
&= 2\left(\frac{2}{15}\right) \\
&= \frac{4}{15}
\end{aligned}$$

- **Correct Option**

  The value of the integral is $\frac{4}{15}$. Hence, the correct option is **(D)**.

www.cuet.pw www.cbse.page

Teaching Guide & Key Concepts:

- **Standard Substitution Strategy for Trigonometric Powers**

  For integrals of the form $\int \sin^m x \cos^n x\,dx$:

(1) If the power of sine (m) is odd positive, substitute $u = \cos x$.

(2) If the power of cosine (n) is odd positive, substitute $u = \sin x$.

(3) This technique works by peeling off one factor to form du, and converting the rest using Pythagoras identities ($\sin^2 x + \cos^2 x = 1$).

- **Properties of Definite Integrals (Symmetry)**

Recognizing symmetry simplifies calculations significantly.

(1) Even Function: If $f(-x) = f(x)$, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

(2) Odd Function: If $f(-x) = -f(x)$, then $\int_{-a}^{a} f(x)\,dx = 0$.

In this problem, the algebraic polynomial $t^2 - t^4$ was even, allowing us to halve the interval and double the result.

- **Short Trick: Wallis' Formula Extension**

There is a direct formula for $\int_{0}^{\pi/2} \sin^m x \cos^n x\,dx$, often called Wallis' formula.

$$I_{\frac{\pi}{2}} = \frac{[(m-1)(m-3)\ldots] \times [(n-1)(n-3)\ldots]}{(m+n)(m+n-2)\ldots} \times K$$

(Where $K = \frac{\pi}{2}$ if both m, n are even, else $K = 1$).

To use this for limit 0 to π:

Check symmetry:

$f(\pi - x) = \sin^3(\pi - x)\cos^2(\pi - x) = \sin^3 x \cos^2 x = f(x)$.

Since $\int_{0}^{2a} f(x)dx = 2\int_{0}^{a} f(x)dx$ if $f(2a - x) = f(x)$, we have:

$$\int_{0}^{\pi} \ldots dx = 2\int_{0}^{\pi/2} \sin^3 x \cos^2 x\,dx$$

Now apply Wallis with $m = 3, n = 2$:

$$2 \times \left[\frac{(3-1)}{(3+2)(3+2-2)(3+2-4)} \times (2-1) \times 1\right] = 2 \times \frac{2 \cdot 1}{5 \cdot 3 \cdot 1} = \frac{4}{15}$$

CUET 2022 PYQ 59

If $\int \left(x+\sqrt{x^2-1}\right)^2 dx = \alpha \cdot x + \beta x^3 + \gamma \left(x^2-1\right)^{3/2} + C$, where C is arbitrary constant, then the value of $3(\alpha+\beta+\gamma)$ is

www.cuet.pw www.cbse.page

(a) 44 (b) 23 (c) 11 (d) 1

Concept Builder

www.cuet.pw www.cbse.page

Teaching Guide & Concept Explanation

- **Algebraic Expansion first:** Before jumping into integration methods like Integration by Parts, check if the integrand can be simplified using basic algebra. The term is of the form $(a+b)^2$. Expanding it often converts a complex composite function into a sum of simple integrable terms.

$$(a+b)^2 = a^2 + b^2 + 2ab$$

- **Integration Techniques Needed:**
  - **Power Rule:** $\int x^n dx = \frac{x^{n+1}}{n+1}$.
  - **Substitution Method:** For terms looking like $x\sqrt{x^2-a^2}$, notice that the derivative of x^2 (which is $2x$) is present outside the root. This suggests the substitution $x^2-a^2=t$.
- **Coefficient Comparison:** Once the integral is solved, the resulting expression must be structurally identical to the one given in the problem. By comparing terms (coefficient of x, x^3, etc.), we can identify the unknowns α, β, γ.

www.cuet.pw www.cbse.page

Detailed Solution

- **Step 1: Expand the Integrand** Let $I = \int \left(x+\sqrt{x^2-1}\right)^2 dx$. Using $(a+b)^2 = a^2+b^2+2ab$:

$$\left(x+\sqrt{x^2-1}\right)^2 = x^2 + (\sqrt{x^2-1})^2 + 2(x)(\sqrt{x^2-1})$$

$$= x^2 + (x^2 - 1) + 2x\sqrt{x^2 - 1}$$
$$= 2x^2 - 1 + 2x\sqrt{x^2 - 1}$$

So, the integral becomes:

$$I = \int (2x^2 - 1 + 2x\sqrt{x^2 - 1})\,dx$$

- **Step 2: Integrate Term by Term** Split the integral into two parts:

$$I = \underbrace{\int (2x^2 - 1)\,dx}_{I_1} + \underbrace{\int 2x\sqrt{x^2 - 1}\,dx}_{I_2}$$

Solving I_1:

$$I_1 = 2\int x^2\,dx - \int 1\,dx = \frac{2x^3}{3} - x$$

Solving I_2: Let $x^2 - 1 = t$. Differentiating both sides: $2x\,dx = dt$.

$$I_2 = \int \sqrt{t}\,dt = \int t^{1/2}\,dt$$
$$I_2 = \frac{t^{3/2}}{3/2} = \frac{2}{3}t^{3/2}$$

Substitute $t = x^2 - 1$ back:

$$I_2 = \frac{2}{3}(x^2 - 1)^{3/2}$$

- **Step 3: Combine and Compare** Combining I_1 and I_2:

$$I = \frac{2}{3}x^3 - x + \frac{2}{3}(x^2 - 1)^{3/2} + C$$

Rearranging to match the given format $\alpha x + \beta x^3 + \gamma(x^2 - 1)^{3/2} + C$:

$$I = (-1)x + \left(\frac{2}{3}\right)x^3 + \left(\frac{2}{3}\right)(x^2 - 1)^{3/2} + C$$

- **Step 4: Identify Coefficients** Comparing the coefficients:
  - ✅ $\alpha = -1$
  - ✅ $\beta = \frac{2}{3}$
  - ✅ $\gamma = \frac{2}{3}$

- **Step 5: Calculate the Required Value** We need to find $3(\alpha+\beta+\gamma)$.

$$\text{Sum} = \alpha+\beta+\gamma = -1+\frac{2}{3}+\frac{2}{3}$$

$$= -1+\frac{4}{3}$$

$$= \frac{-3+4}{3} = \frac{1}{3}$$

Now multiply by 3:

$$3(\alpha+\beta+\gamma) = 3\times\frac{1}{3} = 1$$

- **Conclusion** The value is 1. **Correct Option: (D)**

www.cuet.pw www.cbse.page

Alternative Method (Differentiation)

- **Concept:** Integration is the reverse of differentiation. If $\int f(x)\,dx = G(x)$, then $G'(x) = f(x)$.
- **Differentiate RHS:**

$$\frac{d}{dx}\left[\alpha x+\beta x^3+\gamma(x^2-1)^{3/2}\right]$$

$$= \alpha+3\beta x^2+\gamma\cdot\frac{3}{2}(x^2-1)^{1/2}\cdot(2x)$$

$$= \alpha+3\beta x^2+3\gamma x\sqrt{x^2-1}$$

- **Expand LHS Integrand:**

$$(x+\sqrt{x^2-1})^2 = 2x^2-1+2x\sqrt{x^2-1}$$

- **Compare Terms:**
  - Constant term: $\alpha = -1$
  - x^2 term: $3\beta = 2 \implies \beta = \frac{2}{3}$
  - $x\sqrt{x^2-1}$ term: $3\gamma = 2 \implies \gamma = \frac{2}{3}$
- **Result:**

$$3(\alpha+\beta+\gamma) = 3(-1+2/3+2/3) = 1$$

CUET 2022 PYQ 60

$$\int_0^1 \frac{dx}{x^2+x+1}$$

www.cuet.pw www.cbse.page

(a) $\frac{\pi}{3\sqrt{3}}$ (b) $\frac{\pi}{2\sqrt{3}}$ (c) $\frac{\pi}{6\sqrt{3}}$ (d) $\frac{\pi}{\sqrt{3}}$

Concept Builder

www.cuet.pw www.cbse.page

Detailed Solution and Explanation

- **Concept: Integration by Completing the Square**
  - When the denominator is a quadratic polynomial of the form ax^2+bx+c, and the numerator is a constant (usually 1), we use the method of **Completing the Square**.
  - The goal is to transform the quadratic into the form t^2+a^2 or t^2-a^2.
  - We rely on the standard integral formula:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

www.cuet.pw www.cbse.page

Step-by-Step Calculation

- **Step 1: Complete the Square for the Denominator**
  - Consider the expression: x^2+x+1.
  - To complete the square, take the coefficient of x (which is 1), halve it $(\frac{1}{2})$, and square it $(\frac{1}{4})$. Add and subtract this value.

$$x^2+x+1 = x^2+x+\frac{1}{4}-\frac{1}{4}+1$$

  - Group the perfect square terms:

$$x^2+x+1 = \left(x+\frac{1}{2}\right)^2+\frac{3}{4}$$

- Rewrite $\frac{3}{4}$ as a square:

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

▶ Step 2: Rewrite the Integral

- Substitute the completed square back into the integral:

$$I = \int_0^1 \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

▶ Step 3: Apply the Integration Formula

- Using the standard form $\int \frac{dx}{X^2 + A^2} = \frac{1}{A}\tan^{-1}\left(\frac{X}{A}\right)$.
- Here, $X = x + \frac{1}{2}$ and $A = \frac{\sqrt{3}}{2}$.
- The integral becomes:

$$I = \left[\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left(\frac{x + 1/2}{\sqrt{3}/2}\right)\right]_0^1$$

- Simplify the constants and the argument of $\tan^{-1}$:

$$I = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right]_0^1$$

▶ Step 4: Evaluate the Limits

- **Upper Limit ($x = 1$):**

$$\tan^{-1}\left(\frac{2(1)+1}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right) = \tan^{-1}\left(\sqrt{3}\right)$$

$$\text{Since } \tan\left(\frac{\pi}{3}\right) = \sqrt{3}, \implies \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$

- **Lower Limit ($x = 0$):**

$$\tan^{-1}\left(\frac{2(0)+1}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\text{Since } \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}, \implies \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

Step 5: Final Calculation

- Substitute the limit values back into the expression for I:

$$I = \frac{2}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

- Simplify the fraction inside the bracket:

$$I = \frac{2}{\sqrt{3}}\left(\frac{2\pi - \pi}{6}\right) = \frac{2}{\sqrt{3}}\left(\frac{\pi}{6}\right)$$

- Cancel out the common terms:

$$I = \frac{\pi}{3\sqrt{3}}$$

Final Answer

- The value of the integral is $\frac{\pi}{3\sqrt{3}}$.
- Matches Option **(A)**.

CUET 2022 PYQ 61

Match List I with List II

List - I

A $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$

B $\int_{-\pi/2}^{\pi/2} \sin^2 x \, dx$

C $\int_{0}^{\pi/2} \frac{1}{1+\tan x} dx$

D $\int_{0}^{\pi} |\cos x| \, dx$

List - II

I $\frac{\pi}{2}$

II $\frac{\pi}{4}$

III 0

IV 2

www.cuet.pw www.cbse.page

Choose the correct answer from the options given below:

(A) A-I, B-III, C-II, D-IV

(B) A-III, B-I, C-II, D-IV

(C) A-II, B-III, C-IV, D-I

(D) A-IV, B-III, C-I, D-II

Concept Builder

Step-by-Step Solution

- We will evaluate each integral in List-I one by one and match it with the correct value in List-II.
- **A. Evaluating** $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$
  - Let $f(x) = \sin^7 x$.
  - Check if the function is even or odd:

$$f(-x) = \sin^7(-x) = (-\sin x)^7 = -\sin^7 x = -f(x)$$

  - Since $f(x)$ is an **odd function**, the property of definite integrals states:

$$\int_{-a}^{a} f(x)\, dx = 0 \quad \text{if } f(-x) = -f(x)$$

  - Therefore:

$$\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx = 0$$

  - **Match:** A → III
- **B. Evaluating** $\int_{-\pi/2}^{\pi/2} \sin^2 x \, dx$
  - Let $g(x) = \sin^2 x$.
  - Check if the function is even or odd:

$$g(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x = g(x)$$

  - Since $g(x)$ is an **even function**, the property states:

$$\int_{-a}^{a} g(x)\, dx = 2\int_{0}^{a} g(x)\, dx$$

  - So, the integral becomes:

$$I = 2\int_{0}^{\pi/2} \sin^2 x \, dx$$

  - Using the identity $\sin^2 x = \dfrac{1-\cos 2x}{2}$:

$$I = 2\int_{0}^{\pi/2} \frac{1-\cos 2x}{2}\, dx = \int_{0}^{\pi/2} (1-\cos 2x)\, dx$$

$$I = \left[x - \frac{\sin 2x}{2} \right]_0^{\pi/2}$$

$$I = \left(\frac{\pi}{2} - \frac{\sin \pi}{2} \right) - (0 - 0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

- **Match:** B → I

▷ **C. Evaluating** $\int_0^{\pi/2} \frac{1}{1+\tan x}\, dx$

- Let $I = \int_0^{\pi/2} \frac{1}{1+\tan x}\, dx.$
- Rewrite $\tan x$ as $\frac{\sin x}{\cos x}$:

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\, dx \quad \ldots (1)$$

- Apply the property $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$:

$$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)}\, dx$$

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx \quad \ldots (2)$$

- Adding equations (1) and (2):

$$2I = \int_0^{\pi/2} \frac{\cos x + \sin x}{\cos x + \sin x}\, dx = \int_0^{\pi/2} 1\, dx$$

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

- **Match:** C → II

▷ **D. Evaluating** $\int_0^{\pi} |\cos x|\, dx$

- The function $\cos x$ is positive in $[0, \frac{\pi}{2}]$ and negative in $[\frac{\pi}{2}, \pi]$.

- Split the integral at $x = \frac{\pi}{2}$:

$$I = \int_0^{\pi/2} \cos x \, dx + \int_{\pi/2}^{\pi} (-\cos x) \, dx$$

$$I = [\sin x]_0^{\pi/2} - [\sin x]_{\frac{\pi}{2}}^{\pi}$$

$$I = (\sin\frac{\pi}{2} - \sin 0) - (\sin\pi - \sin\frac{\pi}{2})$$

$$I = (1 - 0) - (0 - 1) = 1 + 1 = 2$$

- **Match:** D → IV

- **Final Matching:**

- A → III
- B → I
- C → II
- D → IV

- Comparing with options, the correct sequence is A-III, B-I, C-II, D-IV.

- **Conclusion:** The correct option is **(B)**.

www.cuet.pw www.cbse.page

Teaching Guide & Key Concepts

- **Odd and Even Functions Property**

- **Property:**

$$\int_{-a}^{a} f(x) \, dx = \begin{cases} 2\int_0^a f(x) \, dx & \text{if } f \text{ is even, i.e., } f(-x) = f(x) \\ 0 & \text{if } f \text{ is odd, i.e., } f(-x) = -f(x) \end{cases}$$

- **Tip:** Always check for this property first when limits are symmetric like $-\pi/2$ to $\pi/2$.

- **King's Property**

- **Property:** $\int_0^a f(x) \, dx = \int_0^a f(a - x) \, dx.$

- **Usage:** Very useful for integrals involving denominators like $1 + \tan x$, $1 + \cot x$, or $\sin x + \cos x$. Adding the original and transformed integrals often cancels the denominator.

- **Modulus Integrals**

- To integrate $|f(x)|$, identify where $f(x)$ changes sign (roots of $f(x) = 0$).
- Split the integration interval at these roots and remove the modulus sign by taking positive or negative signs accordingly.

www.cuet.pw **www.cbse.page**

Short Tricks

Trick for $\int \frac{1}{1+\tan^n x}dx$

- For any integral of the form $\int_0^{\pi/2} \frac{1}{1+\tan^n x}dx$ or $\int_0^{\pi/2} \frac{1}{1+\cot^n x}dx$, the answer is always half the length of the interval.
- Answer $= \frac{1}{2}(\text{Upper Limit} - \text{Lower Limit}) = \frac{1}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4}$.

Trick for Sine Powers

- $\int_{-\pi/2}^{\pi/2} \sin^{\text{odd}} x\,dx = 0$ directly.
- $\int_{-\pi/2}^{\pi/2} \sin^{\text{even}} x\,dx = 2 \times$ (Wallis Formula result).

CUET 2022 PYQ 62

$$\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx =$$

www.cuet.pw www.cbse.page

(A) $\dfrac{e^x}{1+x^2}+C$

(B) $\dfrac{-e^x}{1+x^2}+C$

(C) $\dfrac{e^x}{(1+x^2)^2}+C$

(D) $\dfrac{-e^x}{(1+x^2)^2}+C$

Concept Builder

www.cuet.pw www.cbse.page

Step-by-Step Solution

- **Analyze the Integral Structure:** We are given an integral involving e^x multiplied by a rational function. This strongly suggests the standard form:

$$\int e^x \left[f(x)+f'(x)\right] dx = e^x f(x) + C$$

- **Simplify the Algebraic Part:** Let the term inside the bracket be $g(x)$:

$$g(x) = \left(\frac{1-x}{1+x^2}\right)^2$$

- Expand the numerator $(1-x)^2$:

$$(1-x)^2 = 1-2x+x^2 = (1+x^2)-2x$$

- Substitute this back into the expression:

$$\left(\frac{1-x}{1+x^2}\right)^2 = \frac{(1+x^2)-2x}{(1+x^2)^2}$$

- Split the fraction into two parts:

$$= \frac{1+x^2}{(1+x^2)^2} - \frac{2x}{(1+x^2)^2}$$

$$= \frac{1}{1+x^2} + \left(\frac{-2x}{(1+x^2)^2}\right)$$

- **Identify $f(x)$ and $f'(x)$:** Let us test the first term as $f(x)$.
  - Let $f(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$
  - Differentiate $f(x)$ with respect to x using the chain rule:

$$f'(x) = \frac{d}{dx}(1+x^2)^{-1}$$

$$f'(x) = -1 \cdot (1+x^2)^{-2} \cdot \frac{d}{dx}(1+x^2)$$

$$f'(x) = \frac{-1}{(1+x^2)^2} \cdot (2x)$$

$$f'(x) = \frac{-2x}{(1+x^2)^2}$$

- **Match with the Integral:** The integral can now be rewritten as:

$$I = \int e^x \left[\underbrace{\frac{1}{1+x^2}}_{f(x)} + \underbrace{\left(\frac{-2x}{(1+x^2)^2}\right)}_{f'(x)} \right] dx$$

- **Apply the Formula:** Using $\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + C$, we get:

$$I = e^x \cdot \frac{1}{1+x^2} + C$$

$$I = \frac{e^x}{1+x^2} + C$$

- **Conclusion:** The correct option is **(A)**.

www.cuet.pw www.cbse.page

Teaching Guide & Key Concepts

- **Integration by Parts: The Special Form**

- **Concept:** This is one of the most important forms for competitive exams and boards. Whenever you see e^x multiplying a bracket, immediately check if the terms inside can be split into a function and its derivative.
- **Formula:**

$$\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + C$$

- **Why it works:** If we differentiate $e^x f(x)$, using the product rule:

$$\frac{d}{dx}(e^x f(x)) = e^x f(x) + e^x f'(x) = e^x [f(x) + f'(x)]$$

Integrating both sides brings us back to $e^x f(x)$.

Algebraic Manipulation Skills

- Often, the question is not given directly in the form $f(x) + f'(x)$.
- **Strategy:** 1. Look at the denominator. Try to create the denominator's term in the numerator. 2. In this problem, noticing that $(1-x)^2 = 1 + x^2 - 2x$ was the key step. It allowed us to separate the $(1 + x^2)$ term to cancel one power of the denominator.

www.cuet.pw www.cbse.page

Short Tricks & Methods

Reverse Differentiation (Derivative Check)

- If you are stuck on the integration, use the options!
- Differentiate the options to see which one gives back the integrand.
- **Test Option (A):** $y = \dfrac{e^x}{1 + x^2}$

$$\frac{dy}{dx} = \frac{(1 + x^2)e^x - e^x(2x)}{(1 + x^2)^2}$$

$$\frac{dy}{dx} = e^x \left[\frac{1 + x^2 - 2x}{(1 + x^2)^2}\right]$$

$$\frac{dy}{dx} = e^x \left[\frac{(1-x)^2}{(1 + x^2)^2}\right] = e^x \left(\frac{1-x}{1 + x^2}\right)^2$$

- This matches the question exactly. Hence, (A) is correct. This method is often faster if the algebra in the forward method is tricky.

CUET 2023 PYQ 63

$$\int \frac{\sin\left(\tan^{-1} x\right)}{1+x^2}\,dx =$$

www.cuet.pw www.cbse.page

(A) $-\cos\left(\tan^{-1} x\right)+C$

(B) $\cos\left(\tan^{-1} x\right)+C$

(C) $\tan\left(\cos^{-1} x\right)+C$

(D) $-\tan\left(\cos^{-1} x\right)+C$

Concept Builder

www.cuet.pw www.cbse.page

Step-by-Step Solution

- Given the integral:

$$I = \int \frac{\sin\left(\tan^{-1} x\right)}{1+x^2}\,dx$$

- **Method of Substitution:** We observe that the derivative of $\tan^{-1} x$ is present in the integrand.
- Let us assume:

$$t = \tan^{-1} x$$

- Differentiate both sides with respect to x:

$$\frac{dt}{dx} = \frac{1}{1+x^2}$$

$$dt = \frac{1}{1+x^2}\,dx$$

- **Substitute back into the Integral:** Replacing $\tan^{-1} x$ with t and $\frac{dx}{1+x^2}$ with dt:

$$I = \int \sin(t)\,dt$$

- **Perform the Integration:** We know the standard integral of $\sin t$ is $-\cos t$.

$$I = -\cos t + C$$

- **Back-Substitution:** Replace t with its original value $\tan^{-1} x$:

$$I = -\cos\left(\tan^{-1} x\right) + C$$

- **Conclusion:** Comparing this result with the given options, it matches Option (A).

www.cuet.pw www.cbse.page

Teaching Guide & Key Concepts

Integration by Substitution

- **Concept:** This method is used when the integrand contains a function $g(x)$ and its derivative $g'(x)$.
- **Form:** $\int f(g(x)) \cdot g'(x)\, dx$.
- **Process:** 1. Put $t = g(x)$. 2. Find $dt = g'(x)dx$. 3. The integral simplifies to $\int f(t)\, dt$, which is usually a standard integral.

Key Derivatives to Remember

- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ (This was the key to unlocking this problem).
- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$.

www.cuet.pw www.cbse.page

Short Tricks & Methods

Direct Inspection (Mental Substitution)

- If you recognize the pattern $\int \sin(\text{something}) \cdot (\text{derivative of something})\, dx$, you can treat the "something" as a single variable X in your head.
- Here, "something" is $\tan^{-1} x$. Its derivative $\frac{1}{1+x^2}$ is right there multiplying it.
- So, effectively you are just integrating $\sin(X)$.
- Answer: $-\cos(X) = -\cos\left(\tan^{-1} x\right)$.
- This allows you to solve it in under 10 seconds without writing the 'let $t = \ldots$' steps.

CUET 2022 PYQ 64

$$\int_0^{4\pi} \frac{x}{1+|\cos x|}\,dx =$$

www.cuet.pw www.cbse.page

(a) 4π (b) 16π (c) 32π (d) 64π

Concept Builder

www.cuet.pw www.cbse.page

Step-by-Step Solution

- Let the given integral be I:

$$I = \int_0^{4\pi} \frac{x}{1+|\cos x|}\,dx \quad \ldots(1)$$

- **Step 1: Apply the Property** $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ Here, $a = 4\pi$.

$$I = \int_0^{4\pi} \frac{4\pi - x}{1+|\cos(4\pi - x)|}\,dx$$

- Since $\cos(4\pi - x) = \cos x$, the denominator remains the same:

$$I = \int_0^{4\pi} \frac{4\pi - x}{1+|\cos x|}\,dx \quad \ldots(2)$$

- **Step 2: Add equations (1) and (2)**

$$2I = \int_0^{4\pi} \frac{x + (4\pi - x)}{1+|\cos x|}\,dx$$

$$2I = \int_0^{4\pi} \frac{4\pi}{1+|\cos x|}\,dx$$

$$2I = 4\pi \int_0^{4\pi} \frac{1}{1+|\cos x|}\,dx$$

$$I = 2\pi \int_0^{4\pi} \frac{1}{1+|\cos x|}\,dx$$

- **Step 3: Use Periodicity Property** Let $f(x) = \dfrac{1}{1+|\cos x|}$. Since $|\cos x|$ is periodic with period π, $f(x)$ is also periodic with period π. Using the property $\displaystyle\int_0^{nT} f(x)\,dx = n\int_0^{T} f(x)\,dx$:

$$I = 2\pi \cdot 4 \int_0^{\pi} \frac{1}{1+|\cos x|}\,dx$$

$$I = 8\pi \int_0^{\pi} \frac{1}{1+|\cos x|}\,dx$$

- **Step 4: Use Symmetry Property** Observe that $|\cos(\pi - x)| = |-\cos x| = |\cos x|$. Using the property $\displaystyle\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$ if $f(2a-x) = f(x)$:

$$I = 8\pi \cdot 2 \int_0^{\pi/2} \frac{1}{1+|\cos x|}\,dx$$

$$I = 16\pi \int_0^{\pi/2} \frac{1}{1+\cos x}\,dx$$

(Note: In the interval $\left[0, \dfrac{\pi}{2}\right]$, $\cos x$ is positive, so $|\cos x| = \cos x$).

- **Step 5: Solve the Definite Integral** Use the half-angle formula $1 + \cos x = 2\cos^2\left(\dfrac{x}{2}\right)$:

$$I = 16\pi \int_0^{\pi/2} \frac{1}{2\cos^2\left(\dfrac{x}{2}\right)}\,dx$$

$$I = 8\pi \int_0^{\pi/2} \sec^2\left(\frac{x}{2}\right)\,dx$$

- Integrating $\sec^2\left(\dfrac{x}{2}\right)$:

$$\int \sec^2\left(\frac{x}{2}\right)\,dx = \frac{\tan(x/2)}{1/2} = 2\tan\left(\frac{x}{2}\right)$$

- Applying limits:

$$I = 8\pi \left[2\tan\left(\frac{x}{2}\right)\right]_0^{\pi/2}$$

$$I = 16\pi \left[\tan\left(\frac{\pi}{4}\right) - \tan(0) \right]$$

$$I = 16\pi [1 - 0]$$

$$I = 16\pi$$

▷ **Conclusion:** The correct option is **(B)**.

www.cuet.pw www.cbse.page

Teaching Guide & Key Concepts

- **Removal of 'x' Property (King's Property)**
  - ✓ To solve integrals of the type $\int_0^a x f(x)\,dx$ where $f(a-x) = f(x)$, we use the shortcut:

$$\int_0^a x f(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx$$

  - ✓ This immediately removes the x from the numerator, simplifying the problem to a standard definite integral.
- **Periodicity of Modulus Functions**
  - ✓ The function $|\cos x|$ and $|\sin x|$ have a period of π (not 2π).
  - ✓ Recognizing the period allows you to reduce the upper limit: $\int_0^{n\pi} |\cos x|\,dx = n\int_0^{\pi} |\cos x|\,dx.$
- **Half-Angle Integration Techniques**
  - ✓ Whenever you see $1+\cos x$ or $1-\cos x$ in the denominator, convert them immediately using:
  - ✓ $1 + \cos x = 2\cos^2(x/2)$
  - ✓ $1 - \cos x = 2\sin^2(x/2)$
  - ✓ This converts the denominator into a single term ($\sec^2$ or $\csc^2$), which is easy to integrate.

www.cuet.pw www.cbse.page

Short Trick for Competitions

▷ **Direct Formula Application**

- ✓ If you encounter $\int_0^{n\pi} \frac{x}{1+|\cos x|}\,dx$, remember the steps logic:
- ✓ Step 1: Remove $x \rightarrow$ Factor becomes $\frac{n\pi}{2}$.

- Step 2: Periodicity → Limit reduces to π, multiply by n.
- Integral part $\int_0^{\pi} \frac{1}{1+|\cos x|}\,dx = 2$. (Standard result worth memorizing).
- **Result:** Answer $= \left(\frac{\text{Upper Limit}}{2}\right) \times$ (Value of Integral).
- Here: $\left(\frac{4\pi}{2}\right) \times (4 \times 2)$? No, be careful with n.
- Let's verify: $I = \frac{4\pi}{2} \int_0^{4\pi} \frac{1}{1+|\cos x|}\,dx = 2\pi \times (4 \times 2) = 16\pi$.
- **Fastest Calculation:** $\frac{\text{Limit}}{2} \times 4 \times (\text{Result of } \int_0^{\pi} \ldots) = 2\pi \times 4 \times 2 = 16\pi$.

CUET 2022 PYQ 65

If $\int_0^{\pi/2} \sqrt{\tan x}\, dx = \frac{\lambda}{2}$, then the value of λ is

www.cuet.pw www.cbse.page

(a) π (b) -1 (c) $\frac{\pi}{2}$ (d) $\frac{3}{\sqrt{2}}$

Concept Builder

www.cuet.pw www.cbse.page

Step-by-Step Solution and Explanation

Understanding the Integral

- We are given the definite integral $I = \int_0^{\pi/2} \sqrt{\tan x}\, dx$.
- We need to evaluate this integral and compare it with the given expression $\frac{\lambda}{2}$ to find the value of λ.

Method 1: Algebraic Substitution (Standard Method)

- Let $I = \int_0^{\pi/2} \sqrt{\tan x}\, dx$.
- Make the substitution: Let $\tan x = t^2$.
- Differentiating both sides with respect to x:

$$\sec^2 x\, dx = 2t\, dt$$

$$\left(1 + \tan^2 x\right) dx = 2t\, dt$$

$$\left(1 + t^4\right) dx = 2t\, dt \implies dx = \frac{2t}{1 + t^4}\, dt$$

- Change the limits of integration:
  - When $x = 0$, $t = \sqrt{\tan 0} = 0$.
  - When $x = \frac{\pi}{2}$, $t \to \infty$ (since $\tan\frac{\pi}{2}$ is undefined/infinite).

- Substituting these into the integral:

$$I = \int_0^\infty t \cdot \left(\frac{2t}{1+t^4} \right) dt = \int_0^\infty \frac{2t^2}{1+t^4} dt$$

- Now, we use a standard algebraic manipulation trick. Rewrite $2t^2$ as $(t^2+1)+(t^2-1)$:

$$I = \int_0^\infty \frac{(t^2+1)+(t^2-1)}{1+t^4} dt$$

- Split the integral into two parts:

$$I = \int_0^\infty \frac{t^2+1}{t^4+1} dt + \int_0^\infty \frac{t^2-1}{t^4+1} dt$$

- **Evaluate the first part:** $I_1 = \int \frac{t^2+1}{t^4+1} dt$
  - Divide numerator and denominator by t^2:

$$I_1 = \int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt$$

  - Let $u = t - \frac{1}{t}$, then $du = \left(1+\frac{1}{t^2}\right) dt$.
  - Also, $t^2 + \frac{1}{t^2} = \left(t - \frac{1}{t}\right)^2 + 2 = u^2 + \left(\sqrt{2}\right)^2$.
  - Limits: As $t \to 0$, $u \to -\infty$. As $t \to \infty$, $u \to \infty$.
  - The integral becomes:

$$I_1 = \int_{-\infty}^{\infty} \frac{du}{u^2 + \left(\sqrt{2}\right)^2} = \left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) \right]_{-\infty}^{\infty}$$

$$I_1 = \frac{1}{\sqrt{2}} \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) \right] = \frac{1}{\sqrt{2}}(\pi) = \frac{\pi}{\sqrt{2}}$$

- **Evaluate the second part:** $I_2 = \int_0^\infty \frac{t^2-1}{t^4+1} dt$
  - Divide numerator and denominator by t^2:

$$I_2 = \int_0^\infty \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt$$

* Let $v = t + \frac{1}{t}$, then $dv = \left(1 - \frac{1}{t^2}\right) dt$.
* Limits: As $t \to 0$, $v \to \infty$. As $t \to \infty$, $v \to \infty$.
* Since the lower and upper limits are the same (∞), the value of this definite integral I_2 is **0**.

✅ **Combine results:**

$$I = I_1 + I_2 = \frac{\pi}{\sqrt{2}} + 0 = \frac{\pi}{\sqrt{2}}$$

▶ **Comparing with Given Equation**

✅ We calculated $I = \frac{\pi}{\sqrt{2}}$.

✅ The problem states $I = \frac{\lambda}{2}$.

✅ Equating the two:

$$\frac{\lambda}{2} = \frac{\pi}{\sqrt{2}}$$

$$\lambda = \frac{2\pi}{\sqrt{2}} = \pi\sqrt{2}$$

✅ **Note on Options:** The calculated value is $\pi\sqrt{2}$. Let's check the options: (A) π, (B) -1, (C) $\frac{\pi}{2}$, (D) $\frac{3}{\sqrt{2}}$.

✅ None of the options exactly match $\pi\sqrt{2}$. However, in many textbook problems of this type, there is often a typo in the question statement.

* Case 1: If the question meant $\int_0^{\pi/2} \sqrt{\tan x}\, dx = \frac{\lambda}{\sqrt{2}}$, then $\lambda = \pi$ (Option A).
* Case 2: If the question meant $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}}\, dx = \frac{\lambda}{2}$, then the integral is $\frac{\pi}{4}$, so $\frac{\pi}{4} = \frac{\lambda}{2} \implies \lambda = \frac{\pi}{2}$ (Option C).

✅ Based strictly on the image provided, the mathematical answer is $\lambda = \pi\sqrt{2}$.

www.cuet.pw www.cbse.page

Method 2: Gamma Function (Short Trick)

▶ This method is much faster for competitive exams.

- **Formula:**

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$$

- Rewrite the integral:

$$\sqrt{\tan x} = \sqrt{\frac{\sin x}{\cos x}} = \sin^{1/2} x \cdot \cos^{-1/2} x$$

  Here, $m = \frac{1}{2}$ and $n = -\frac{1}{2}$.

- Substitute into the formula:

$$I = \frac{\Gamma\left(\frac{1/2+1}{2}\right)\Gamma\left(\frac{-1/2+1}{2}\right)}{2\Gamma\left(\frac{1/2-1/2+2}{2}\right)} = \frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)}{2\Gamma(1)}$$

- Use Euler's Reflection Formula $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin(n\pi)}$. Let $n = \frac{1}{4}$, so

$$\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \frac{\pi}{\sin\left(\frac{\pi}{4}\right)} = \frac{\pi}{1/\sqrt{2}} = \pi\sqrt{2}.$$

- Calculate I:

$$I = \frac{\pi\sqrt{2}}{2(1)} = \frac{\pi}{\sqrt{2}}$$

- This confirms our previous result.

Concept Builder

www.cuet.pw www.cbse.page

Problem Statement

Given the integral:

$$\int_0^{\pi/2} \sqrt{\tan x} \, dx = \frac{\lambda}{2} \tag{1}$$

Find the value of λ.

www.cuet.pw www.cbse.page

Solution

Let the integral be I.

$$I = \int_0^{\pi/2} \sqrt{\tan x}\, dx \tag{2}$$

www.cuet.pw www.cbse.page

Step 1: Substitution

Let $\tan x = t^2$. Differentiating both sides:

$$\sec^2 x\, dx = 2t\, dt$$
$$(1 + \tan^2 x)\, dx = 2t\, dt$$
$$(1 + t^4)\, dx = 2t\, dt \implies dx = \frac{2t}{1 + t^4} dt$$

www.cuet.pw www.cbse.page

Step 2: Change Limits

- When $x = 0$, $t = \sqrt{\tan 0} = 0$.
- When $x \to \pi/2$, $t = \sqrt{\tan(\pi/2)} \to \infty$.

Substituting these into the integral:

$$I = \int_0^{\infty} t \cdot \frac{2t}{1 + t^4} dt = \int_0^{\infty} \frac{2t^2}{1 + t^4} dt \tag{3}$$

www.cuet.pw www.cbse.page

Step 3: Algebraic Manipulation

Rewrite the numerator $2t^2$ as $(t^2 + 1) + (t^2 - 1)$:

$$I = \int_0^{\infty} \frac{t^2 + 1}{1 + t^4} dt + \int_0^{\infty} \frac{t^2 - 1}{1 + t^4} dt \tag{4}$$

www.cuet.pw www.cbse.page

Evaluating Part A:

$$\int_0^\infty \frac{t^2+1}{t^4+1}\,dt = \int_0^\infty \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\,dt = \int_0^\infty \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+2}\,dt$$

Let $u = t - \frac{1}{t}$, so $du = \left(1+\frac{1}{t^2}\right)dt$. Limits become $-\infty$ to ∞.

$$= \int_{-\infty}^{\infty} \frac{du}{u^2+(\sqrt{2})^2} = \left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)\right]_{-\infty}^{\infty}$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \frac{\pi}{\sqrt{2}}$$

www.cuet.pw www.cbse.page

Evaluating Part B:

$$\int_0^\infty \frac{t^2-1}{t^4+1}\,dt = 0 \quad \text{(Evaluating this integral yields zero)}$$

www.cuet.pw www.cbse.page

Step 4: Solve for λ

The total value of the integral is:

$$I = \frac{\pi}{\sqrt{2}} \tag{5}$$

Equating this to the given expression:

$$\frac{\pi}{\sqrt{2}} = \frac{\lambda}{2}$$

$$\lambda = \frac{2\pi}{\sqrt{2}}$$

$$\lambda = \pi\sqrt{2}$$

www.cuet.pw www.cbse.page

Answer

The value of λ is $\pi\sqrt{2}$.

CUET UG JEE Mains NEET UG NCET

All these important exams is conducted by NTA. Very often some questions are twisted and repeated from these exams. So we include all the recent PYQs of all these exams to strenghten your CUET UG preparation.

Practicing these exam questions keeps you more relevant according to the question pattern of NTA.

NCET and CUET UG has same question pattern. Only difference is in the total number of questions being asked per subject.

Never buy or waste money on any mock test series because many mock test series are either self-made questions or old repeated or twisted PYQs of these exams or other entrance exams. Depending on those MOCK TEST SERIES may help you to practice questions but may keep you unprepared according to NTA's question pattern.

We have given some chapters for free to let you see what value are we providing you.

For all chapter pyqs question bank, please install our app from our website

www.cuet.pw

then purchase our question bank. Please support our hard work and effort. Please !!!!

Also check our Physics and Chemistry or others exam and subjects question bank.

Telegram Links

Just Search on Telegram

- @cuetpyq
- @books_cuet
- @class_12th_pyq
- @jee_cbse
- @pyqs12
- @cbsegroup
- @board_cbse

For CBSE Class 12 board exam chapter wise pyq question bank go to

www.cbse.page